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STANDARDIZATION OF TAKE-OFF PERFORMANCE MEASUREMENTS FOR AIRPLANES

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SUMMARY:

Formulae are derived by which take-off performance measurements may be standardized. These formulae are easy to use and they apply, with suitable numerical constants, to airplanes with any type of propulsive system, including mixed types and types using part-time assistance or boost.

Numerical constants are proposed for use with these formulae. Experimental data available support these constants, but are insufficient to check them completely.

NOTATION:

Symbol	Definition
C_Q	$Q/\rho n^2 d^5$
C_T	$F/\rho n^2 d^4$
d	Propeller diameter
D	Total resistance at Speed V (aerodynamic drag + rolling drag)
$\overline{D}$	Value of D at speed $\overline{V}$
F	Total thrust at speed V
$\overline{F}$	Total thrust at speed $\overline{V}$
F_o	Static thrust
$\overline{F}_j$	Jet thrust of turbo propeller engine at speed $\overline{V}$
$\overline{F}_p$	Propeller thrust at speed $\overline{V}$
F_R	ATO thrust
$\overline{F}_R$	Mean effective ATO thrust
$\overline{F}_b$	Thrust of basic power plants (without ATO) at speed $\overline{V}$
h	Height of airplane above runway
h_v	$(V_{50}^2 - V_T^2)/2g$
J	V/nd

Symbol	Definition
n	Propeller speed
N	Engine speed
P	Power input to propeller
P_a	Ambient air pressure
Q	Torque input to propeller
$\mathcal{R}$	$\overline{F}/\overline{F}_R$
S_a	Distance from take-off to 50 feet
S_g	Length of ground roll
S_{g_w}	Ground roll with headwind
S_{g_o}	Ground roll corrected to zero wind
t_a	Time in air phase
t_{R_a}	Time of ATO operation in air phase
t_{R_g}	Time of ATO operation in ground phase
V	True ground speed
V_T	Value of V at take-off
V_{50}	Value of V at 50 feet
$\overline{V}$	Mean speed during phase (ground roll or air phase)
W	Airplane gross weight
w	Headwind
γ_c	Angle of steady climb
δ	Ambient air pressure/standard sea level pressure
θ	Air temperature $^{o}K/288$
σ	Air density/standard sea level air density
$\sin\phi$	Slope of runway (positive for uphill)

Subscripts "s" and "t" refer to standard and test conditions respectively. The prefix Δ indicates the correction required to bring the test value of the parameter to standard.

INTRODUCTION:

1. In the presently used method of standardizing the take-off performance of piston engined airplanes it is assumed that the excess of thrust power over drag power on or near the ground is equal to (or at least proportional to) that in the air. Also, it is assumed that during the air phase of the take-off this excess thrust power is used only to raise the airplane to 50 feet, the usual increase in kinetic energy between the takeoff and 50 feet being neglected. With increases in loading and in take-off speeds, both of these assumptions have become untenable.

A revised method is developed in this report which does not require the above assumptions. The method is readily applied to airplanes with propeller, jet or mixed propulsion systems.

FACTUAL DATA:

2. Basic Relations:

2.1 Ground Roll: The equation of motion for take-off from a level runway in zero wind may be written:

$$Sg = \int_0^{S_g} dS$$

$$= \int_0^{V_T} \left\{ \frac{dS}{dt} \Big/ \frac{dV}{dt} \right\} dV$$

$$= \frac{W}{g} \int_0^{V_T} \frac{V\,dV}{F - D} \qquad (2\text{-}1)$$

where D = Total resistance, including tire friction

F = Total net thrust

S = Distance from start of run to speed V

Sg = Distance to unstick

t = Time

V = True speed

V_T = True speed at unstick

(Corrections for wind and runway slope are considered later.)

To adjust the observed performance to standard conditions we wish to know how Sg will vary with air temperature and pressure, airplane gross weight and net thrust. To do so directly from equation (2-1), would usually be tedious. Fortunately, for purposes of performance reduction it is permissible (and customary) to approximate and to work with a "mean excess thrust", which would, if applied throughout the run, give the same ground roll. To reach a speed V_T in a distance Sg with a constant acceleration would require an acceleration of $V_T^2/2$ Sg. The mean excess thrust is therefore given by:

mean excess thrust $= WV_T^2/2g\ Sg$ (2-2)

The actual excess thrust changes steadily as the speed increases, so the mean thrust will be equal to the actual excess thrust at some speed V. Then if $\overline{F}$ and $\overline{D}$ are the thrust and total resistance at $V = \overline{V}$

$$S_g = \frac{W}{2g} \quad \frac{V^2_T}{\overline{F} - \overline{D}} \tag{2-3}$$

It is showh in Appendix I, that a close approximation to $(\overline{F} - \overline{D})$ is obtained by assuming that:

$$\overline{V} = 0.75\ V_T$$

With this assumption, the effect on take-off ground roll of changes in test conditions may be deduced from equation (2-3) and the associated changes in W, V_T, $\overline{F}$ and $\overline{D}$.

2.2 Air Phase: The equation of motion for the air phase with zero wind may be written:

$$S_a = \int_0^{S_a} ds$$

$$= \int_{V_T}^{V_{50}} \left\{ \frac{ds}{dt} \Big/ \frac{dE}{dt} \right\} dE \tag{2-4}$$

where S = Distance from unstick point

S_a = Distance from unstick to 50 feet

t = Time

E = Total energy of the airplane relative to its energy at the start of the ground roll

Now $$E = W\left(h + \frac{V^2}{2g}\right) \tag{2-5}$$

where h = height above start of ground roll

also $$\frac{dE}{dt} = V\ (F-D) \tag{2-6}$$

Substituting from (2-6) into (2-4)

$$S_a = \int \frac{V\,dE}{(F-D)V} = W \int \frac{d\left(h + \frac{1}{2}\frac{V^2}{g}\right)}{F - D} \tag{2-7}$$

As with the ground roll, we may approximate, writing

$$S_a = \frac{W(50 + h_v)}{\overline{F} - \overline{D}} = \frac{50\,W}{\overline{F} - \overline{D}} \quad \frac{50 + h_v}{50} \tag{2-8}$$

where $\overline{F}$, $\overline{D}$ are mean values of F and D and h_v is $(V_{50}^2 - V_T^2)/2g$.

It is not possible, in this case, to be very precise about the conditions under

which the actual thrust and drag will be equal to the mean values. However, the speed range is not great and its effect on $\bar{F}$ and $\bar{D}$ will be slight. It will be assumed that $\bar{F}$ and $\bar{D}$ are equal to the values of F and D at the speed attained at 50 feet (in the classical take-off with a climb away at constant speed, this is the steady climb speed) and a height of 25 feet. It is of interest that on large airplanes the ground effect may persist fairly strongly even at 50 feet. However, this need only be considered in setting up the method. It need not usually be considered in its routine application.

3. Speed at 50 Feet Altitude:

It is clear from equation (2-8) that the effect on air distance of any parameter such as air density will be strongly influenced by any change of h_v with that parameter. Let us examine the relation between take-off speed and the speed at 50 feet.

If the airplane leaves the ground at the maximum safe lift coefficient it has at that instant insufficient lift available to change the direction of flight. It will, however, continue to accelerate and as it does so, the available lift will increase and enable the airplane to change its direction of flight, say to that at which it can climb at constant forward speed. However, unless the remainder of the climb to 50 feet is a zoom with speed decreasing, some of this speed increase will necessarily remain at 50 feet. If the unstick is delayed to a speed higher than the minimum the airplane can be brought up into the climb more sharply, and the speed change lessened, but even then a fairly drastic maneuver would be necessary to bring the speed at 50 feet back to the unstick speed. Such a maneuver would be unusual except with light and docile airplanes.

Present take-off speeds for high performance airplanes are high enough that this change in kinetic energy between unstick and 50 feet is between 60% and 70% of the total energy increase, only 30% to 40% being used to increase altitude. It is, therefore, necessary to decide how great the increase in kinetic energy is and how it varies with test conditions.

As a matter of interest, and to fix ideas, let us first consider the type of take-off in which unstick and transition are at constant lift coefficient and the remainder (if any) of the climb to 50 feet is a straight climb at constant speed. It is shown in Appendix III from an approximate analysis that in such a case:

$$V_{50} = V_T \left(1 + \frac{\gamma_c}{\sqrt{2}}\right) \tag{3-1}$$

where γ_c is the angle of steady climb at the eventual steady climb speed V_{50}. The total horizontal distance is covered then (Appendix III)

$$S_a = \frac{50}{\gamma_c} + \frac{V_T^2}{2g} \tag{3-2}$$

If, as happens when the ratio of thrust to weight is large, the curved part of the flight path is not completed at 50 feet, a more complicated relation gives the speed of 50 feet. The value of $h_v / h_v + 50$ given by this type of take-off is shown by the full curve of Figure 1A. The straight portion refers to these cases for which the transition is complete before 50 feet, the curved portion to those for which the airplane is still on a curved path at 50 feet. The chain

dashed line is an empirical adaptation of the theory, derived by the British around 1940 and applied to take-offs for which the aim was to achieve "the minimum distance compatible with safety". As an illustration, the ratio of the speed at 50 feet to the speed at take-off, has been computed from a number of test take-offs and is plotted in Figure 2 against the available angle of climb (i.e. the excess of thrust over drag, divided by the gross weight). It will be seen that the trend to more increase in speed between take-off and 50 feet at higher values of γ_c is present, not only for the points as a whole, but also for most of the individual airplanes. However, no definite trend is apparent for medium propeller airplane No. 2. In the case of the airplane with fixed pitch propeller, most of the take-offs were ended by zoom climbs to 50 feet, and a subsequent reduction of the attitude to that for a steady climb, to achieve the minimum distance. Such a maneuver could hardly be considered practical except on airplanes of that class with a gentle stall. The two take-offs by that airplane using a more normal technique tend to agree with the other airplanes. The data as a whole, tend to follow the British empirical relation.

If the British relation between V_{50}/V_{TO} and γ_c were accepted instead of the presently used assumption of constant lift coefficients at take-off and at 50 feet it would markedly reduce the effect of changes in thrust, density, or weight, on the length of the air phase. For example, if the standard value of γ_c were 90% of the test value due to thrust changes, the standard speed increase between take-off and 50 feet would thereby be reduced to 90% of the test value. For an h_v of 100 feet, this would result in a change in air distance to (equation 2-8):

$$\frac{\gamma_{c_t}}{\gamma_{c_s}} \quad \frac{50 + h_{v_s}}{50 + h_{v_t}} \times S_{a_t}$$

$$= \frac{1}{0.9} \quad \frac{50 + 90}{50 + 100} \quad S_{a_t} = 1.04\, S_{a_t}$$

whereas the assumption of constant lift coefficients at take-off and at 50 feet would give

$$\frac{\gamma_{c_t}}{\gamma_{c_s}}\, S_{a_t} = \frac{1}{0.9}\, S_{a_t} = 1.11\, S_{a_t}$$

The present writer has the opinion that the British empirical relation was a good assumption when it was made, and that the basic trend which it represents still exists. There are a number of factors, however, of increasing importance, which tend to reduce its present suitability. The older tail wheel type of landing gear never forced a delayed take-off, whereas tricycle landing gears sometimes do so and "bicycle" landing gears leave the pilot little control over unstick speed. Also, with modern airplanes operating from good runways, it is likely to be economic to delay unstick and use the excess speed to achieve a sharper transition and a shorter air path, because the drag on the ground will often be lower than that in the air. With bicycle landing gear in particular, it would seem reasonable to trim before take-off for a desired climb lift coefficient. The airplane would then unstick when aerodynamic forces were sufficient either to lift the airplane in the ground attitude or to pitch it to a greater attitude sufficient to lift it. Cases such as these, taken with the increasing need to rely on the airspeed indicator, rather than the "feel" of the airplane as

data by which to judge how to take off, suggest that an assumption that the lift coefficients at 50 feet, and at unstick, are constant during the reduction process, is probably the most reasonable to use. It enables the quoted take-off performance at a given gross weight, to be associated with a specific speed at 50 feet, which will be independent of air pressure and temperature. This assumption will, therefore, be adopted for this Report.

4. Performance Reduction Equations:

4.1 Ground Roll: Using equation (2-3) we may consider the effect on S_g of moderate changes in air density, thrust, and airplane gross weight. This may be done by using the equation directly or by differentiating it with respect to our variables. Let us first consider direct use of the equation. It is shown in Appendix II that if unstick is at constant C_L

$$\frac{S_{g_s}}{S_{g_t}} = \frac{W_s}{W_t}\,\frac{\sigma_t}{\sigma_s} \Big/ \left\{ \frac{2g\,S_{g_t}}{W\,V_{r_t}^2}\left(\frac{W_t}{W_s}\,\overline{F}_s - \overline{F}_t\right) + 1 \right\} \qquad (4\text{-}1)$$

where subscripts "t" and "s" refer to test and standard conditions respectively. This form is intended primarily for machine computing of the corrected take-off performance of jet propelled airplanes. It requires evaluation of the test and standard thrust, which presents no great difficulty with the jet airplane but could be inconvenient with a propeller driven airplane. It would be a little clumsy for desk computing, but it has the advantage of being more accurate for large corrections than a differential method. As F varies only slowly with air-speed, it is usually sufficient to write $\overline{F} = 0.94 \times F_O$, the static thrust. Alternatively, we may differentiate equation (2-3) with respect to our variables. Assuming again that take-off is at constant C_L, we have V_T^2 proportional to W/σ and hence

$$S_g \propto \frac{W^2}{2g\sigma}\left(\frac{1}{\overline{F} - \overline{D}}\right) \qquad (4\text{-}2)$$

This being so, it is shown in Appendix II that

$$\frac{\Delta S_g}{S_{g_t}} = \frac{\Delta W}{W_t}\left\{2 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right\} - \frac{\Delta\sigma}{\sigma_t} - \frac{\Delta\overline{F}}{\overline{F}_t}\left\{1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right\} \qquad (4\text{-}3)$$

where the terms in ΔW and $\Delta\sigma$ give the effect of W and σ on S_g at constant $\overline{F}$. Changes in W and σ will also, in general, affect S_g by changing $\overline{F}$, but any such effects will be accounted for in estimating $\Delta\overline{F}$. If weight corrections are small, it would be convenient and legitimate to substitute W_s for W_t above.

This relation is general, being independent of the particular method of propulsion. It is quite convenient to use, if, as seems likely, generalized values of $\overline{D}/(\overline{F}-\overline{D})$ can be used. This and the estimation of $\Delta\overline{F}/\overline{F}_t$ for propeller driven airplanes are considered later.

An alternative form of equation (4-3), derived in Appendix II is

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2 + \frac{\overline{D}}{\overline{F} - \overline{D}}} \frac{\sigma_t}{\sigma_s} \left(\frac{\overline{F}_t}{\overline{F}_s}\right)^{1 + \frac{\overline{D}}{\overline{F} - \overline{D}}} \qquad (4\text{-}3a)$$

This form would be a little more accurate for large corrections, being equivalent to a step by step application of equation (4-2).

4.2 Air Phase: For jet airplanes when mechanized computing is used, or possibly when the departure of test conditions from standard is rather large, it is profitable to use a relation similar to equation (4-1). It is shown in Appendix II, that if the lift coefficients at unstick and at 50 feet are unchanged during the reduction process, we may write

$$\frac{S_{a_s}}{S_{a_t}} = \frac{\frac{W_s}{W_t}\frac{\sigma_t}{\sigma_s}h_{v_t} + 50}{h_{v_t} + 50 + \frac{S_{a_t}\bar{F}_s}{W_s} - \frac{S_{a_t}\bar{F}_t}{W_t}} \qquad (4\text{-}4)$$

where S_a is the distance covered between unstick and 50 feet, and, again, subscripts "t" and "s" refer to test and standard conditions respectively.

For propeller driven airplanes, and in general for desk computing, when the differences between test and standard conditions are not too large, it is again more convenient to work with an equation derived by differentiating the basic relation. It is shown in Appendix II that:

$$\begin{aligned}\frac{\Delta S_a}{S_{a_t}} = {} & \frac{\Delta W}{W_t}\left\{1 + \frac{\bar{D}}{\bar{F}-\bar{D}} + \frac{h_v}{h_v + 50}\right\} \\ & - \frac{\Delta\bar{F}}{\bar{F}_t}\left\{1 + \frac{\bar{D}}{\bar{F}-\bar{D}}\right\} \\ & - \frac{\Delta\sigma}{\sigma_t}\left\{\frac{h_v}{h_v + 50}\right\}\end{aligned} \qquad (4\text{-}5)$$

Again, as with equation (2-6), the terms in ΔW and $\Delta\sigma$ give the change of S_a with W and σ at constant $\bar{F}$; any effect of changes of W or σ on $\bar{F}$ must be included in $\Delta\bar{F}$. As with the ground roll case, the relation is general, being independent of the method of propulsion used. The relation is quite easy to apply, provided that $\Delta\bar{F}/\bar{F}_t$ and the values of h_v and $\bar{D}/(\bar{F}-\bar{D})$ are readily estimated. These matters are considered later.

Equation (2-3), as (2-6), can also be written in exponential form

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{1 + \frac{\bar{D}}{\bar{F}-\bar{D}} + \frac{h_v}{h_v+50}} \left(\frac{\sigma_t}{\sigma_s}\right)^{\frac{h_v}{h_v+50}} \left(\frac{F_t}{F_s}\right)^{1 + \frac{\bar{D}}{\bar{F}-\bar{D}}} \qquad (4\text{-}5a)$$

This form, like equation (2-6a), is a little more accurate for large corrections.

5. Ratio of Mean Drag to Mean Excess Thrust:

If, in either the ground or air phase, $\bar{F}$ is readily estimated, then the ratio of drag to excess thrust required to apply equation (4-3) and (4-5), may be deduced from the relations

$$\frac{\overline{D}}{\overline{F} - \overline{D}} = \frac{\overline{F}}{\overline{F} - \overline{D}} - 1 \qquad (5\text{-}1)$$

and $$\overline{F} - \overline{D} = WV_T^2/2g \; S_g \qquad (5\text{-}2)$$

or $$\overline{F} - \overline{D} = W\,(h_V + 50)/S_a$$

for the ground roll or air phase respectively.

This is what, in effect, is done in deriving equations (4-1) and (4-4). However, manual computing would be greatly simplified by assuming generalized values for $\overline{D}/(\overline{F} - \overline{D})$ for ground and air phases. This seems to be practicable. Table I below, lists estimated values of this ratio for a number of airplanes, based on values of $(\overline{F} - \overline{D})/W$ computed from test data and estimated values of $\overline{D}/W$ (for propeller airplanes) or $\overline{F}/W$ (jet airplanes). (An exception is the heavy propeller airplane, for which the values are based on the firm's estimates of performance.)

TABLE I

Airplane Type	Ratio of Drag to Excess Thrust: Ground Roll	Ratio of Drag to Excess Thrust: Air Phase
Light, fixed pitch (Zero Flap)	0.2	0.3
Light, fixed pitch (50° Flap)	0.5	0.8
Light, constant speed (15° Flap)	0.2	0.5
Light, constant speed	0.3	0.5
Heavy, propeller	0.2	0.6
Medium, propeller	0.3	0.6
Medium, jet*	0.5	1.2 Approx.
Medium, jet	0.3	0.9 Approx.
Fighter, jet	0.3	0.9

* Take-off accelerations very poor for the class of airplane.

It should be remarked that the estimates for the air phase are rather rough, because the experimental values of $(\overline{F} - \overline{D})$ were usually very erratic. This is a common feature of take-off data, which very probably results from inaccuracies in the measurement of the airplane speeds at take-off and at 50 feet. This difficulty is, of course, an argument against the direct use of experimental speeds (as in equation (4-4), where they are used to calculate h_V).

On the basis of Table I, it is suggested that values of 0.3 for the ground run and 0.6 for the air phase be used. These would appear to be representative values except when the acceleration is very poor, (say less than 0.1 g during

the ground run) when almost any correction method is liable to give results. In such cases, particular care should be taken either to run the tests under conditions as near as possible to standard, or to test under a range of conditions wide enough to give a check of the reduction formulae.

6. Ratio of Kinetic to Total Energy Increase on Climb:

To apply equation (4-5), it would also be very convenient to have generalized values for $h_v/(h_v + 50)$, that is, the ratio of the kinetic to the total energy increase on the climb.

In Figure 1A experimental values of $h_v/(h_v + 50)$ are plotted for a wide range of types of airplane. It will be seen, as experience of take-offs would have suggested, that the scatter is very large. It should be noted that it was difficult to raise the nose wheel of the jet fighter in certain configurations. The effect of this is illustrated in Figure 1B which presents records of two take-offs of this airplane made at approximately the same gross weight. In the one case, the unstick speed was slow, and the transition long, persisting almost to 50 feet. This, in fact, agrees well with the theory outlined above. In the second case, the unstick speed was much higher, the transition brief, and the climb-away quite steep. These are typical of what happens with early unstick and late unstick, respectively. It would seem, for a given speed at 50 feet,that the latter may well become the more normal, and probably, the more economical type of maneuver.

Equation (4-4) implies that h_{v_t} is computed from the test data. This is probably the most satisfactory method if precision is desired, despite the difficulty of measuring V_T and V_{50} accurately. However, if desired, a rather rough estimate of h_v could be made from generalized data. Similarly, when using equation (4-5), computation of the test value of h_v is probably desirable from the point of view of accuracy. The weight error correction is usually small, so the decision will be determined mainly by the size of the term $\Delta\sigma/\sigma_t$. For example, if test conditions were 2000 feet pressure altitude and 35°C and standard conditions were sea level 15°C $\Delta\sigma/\sigma_t$ would be 0.13. An error of 0.3 in the value assumed for $h_v/h_v + 50$ would then lead to an error of 4% in the corrected air distance. If this order of accuracy is sufficient for correction over so large a range, as it may well be in many cases, then manual computation of the correction could be simplified by assuming, say

$$\frac{h_v}{h_v + 50} = 0.4 \text{ for light airplanes}$$

$$\frac{h_v}{h_v + 50} = 0.7 \text{ for other airplanes}$$

The figure of 0.7 is a little on the high side for present airplanes and allows for some future increase in ratio. When machine computing is used the simplification has little value.

7. Thrust Correction:

7.1 General: The above equations give the corrections to be added to

the test distances in terms of the corrections required to bring the test weight, air density, and thrust to standard. These equations are independent of the particular method of propulsion and can be applied to airplanes having propeller, turbojet, rocket or mixed propulsion systems provided all of the systems operate throughout the take-off. Assistance which operates during only part of the take-off is considered separately later in the report.

7.2 Jet Propulsion: Estimation of test and standard thrusts for a jet airplane is fairly straight forward and little general comment can usefully be made.

Rocket thrust does not vary with speed during take-off and the thrust of the turbojet engine does not vary rapidly. Consequently, it is unnecessary to correct the test thrust for the changes in mean speed between test and standard conditions. With the turbojet engines, however, it is desirable to estimate the test and standard thrusts at approximately the right speeds. It is best to base these estimates on a measured test thrust and an estimated correction, but failing this, the mean thrusts may be assumed to be, say, 94% of static thrust if the air intake pressure losses under static conditions are not large. However, some care is necessary; for example use of an intake designed for very high speed may result in a thrust which is relatively poor under static conditions but recovers during take-off as the velocity ratio (inlet speed/free air speed) becomes smaller. The engineer must use his discretion in such cases.

7.3 Fixed Pitch Propellers: Fixed pitch propellers are presently found only on very light low speed airplanes with unsupercharged engines. For such airplanes it is proposed to approximate the reaction of the propeller to change in operating conditions by assuming that:

a. The torque coefficient C_Q is unchanged between test and standard conditions

b. The thrust coefficient C_T varies linearly with advance diameter ratio J, the slope being such that C_T becomes zero at take-off rpm at an airspeed equal to five times the take-off speed

where
$$C_Q = \text{torque}/\rho n^2 d^5$$
$$C_T = \text{thrust}/\rho n^2 d^4$$
$$J = V/nd$$
n = propeller rotational speed

d = propeller diameter

The engine may be at maximum permissible speed or at full throttle. In the first case, reduction will be at constant engine speed and we will have

$$\text{thrust} \propto \sigma \, C_T$$
$$\frac{dF}{F} = \frac{d\sigma}{\sigma} + \frac{dC_T}{C_T} \qquad (7\text{-}1)$$

It is shown in Appendix IV that with the above assumptions we may then write:

$$\frac{\Delta F}{F_t} = 1.1 \frac{\Delta\sigma}{\sigma_t} - 0.1 \frac{\Delta W}{W_t} \qquad (7\text{-}2)$$

In the second case, of the full throttle engine, we must also make assumptions about the variation of available torque with air pressure and air temperature. It is proposed to assume that

$$\text{torque available} \propto \frac{P_a}{\sqrt{T_a}}$$

With this assumption, as C_Q is to be constant,

$$\frac{P_a}{\sqrt{T_a}} \propto \rho n^2 \propto \frac{P_a}{T_a} n^2$$

that is,

$$n \propto T_a^{\frac{1}{4}}$$

We then have (Appendix IV)

$$\frac{\Delta F}{F_t} = 1.1 \frac{\Delta\sigma}{\sigma_t} + 0.4 \frac{\Delta T_a}{T_{a_t}} - 0.1 \frac{\Delta W}{W_t} \qquad (7\text{-}3)$$

$$\frac{\Delta N}{N_t} = \frac{1}{4} \frac{\Delta T_a}{T_{a_t}} \qquad (7\text{-}4)$$

7.4 Constant Speed Propellers: It is shown in Appendix IV that change of propeller thrust between test and standard conditions may be written, in the absence of compressibility effects on efficiency:

$$\frac{\Delta F}{F_t} = A_P \frac{\Delta P}{P_t} + A_\sigma \frac{\Delta\sigma}{\sigma_t} + A_N \frac{\Delta N}{N_t} + A_W \frac{\Delta W}{W_t} \qquad (7\text{-}5)$$

where A_p, A_σ, A_n, and A_W are propeller functions. Generalized curves for these functions in terms of propeller efficiency η and advance diameter ratio J are given in Figure 3. To use the figure an approximate estimate of η would be made by comparing the power input to the propellers with the thrust power as given by the observed acceleration or climb and the estimated drag.

Precise values of these functions are not required, it being sufficient to estimate them within 0.1 or 0.2. Further generalization is, therefore, attractive. To study this the positions representing the mean conditions during the ground roll and the air phase have been estimated and mapped on the diagrams. (These points are not, of course, experimental data. They merely indicate the position of the particular case on the diagram). From inspection of these points it is tentatively proposed to assume that

$$A_P = 0.7$$

$$A = 0.5$$

$$A_N = +0.5$$

$$A_W = -0.2$$

These numbers are tentative only, and should be checked from time to time. In case of doubt, however, one may revert to the curves. Corrections for engine speed will usually be very small, so a relatively large error in A_N may be tolerated.

7.5 Mixed Systems, Including ATO and Turbo Propellers: Mixed propulsion systems are quite straight forward to deal with when all components are operating throughout the take-off and climb-away. However, one must know what proportion of the total mean thrust is contributed by each group. For example, suppose that an airplane had both propellers and turbojet engines; then we may write

$$\frac{\Delta \bar{F}}{\bar{F}} = \frac{\Delta \bar{F}_j}{\bar{F}} + \frac{\Delta \bar{F}_p}{\bar{F}_p} \frac{\bar{F}_p}{\bar{F}} \qquad (7\text{-}6)$$

where $\bar{F}_j$ = thrust from jet engines

$\bar{F}_p$ = thrust from propellers

$\bar{F} = \bar{F}_j + \bar{F}_p$

and $\Delta \bar{F}_p / \bar{F}_p$ may be estimated as in the preceding section.

The turbo propeller engine, of course, is usually mixed installation from this aspect, as there is usually a small but appreciable residual jet thrust, amounting to about 10% of the total. However, experience may show that in this case it may be accurate to assume that the jet and propeller thrust are in constant proportion, so that

$$\frac{\Delta \bar{F}}{\bar{F}} = \frac{\Delta \bar{F}_p}{\bar{F}_p} \qquad (7\text{-}7)$$

8. Preliminary Corrections:

8.1 Correction of Ground Roll For Wind: Correction of the ground roll for wind is presently made by the formula

$$\frac{S_{g_0}}{S_{g_w}} = \left(1 + \frac{w}{V_r}\right)^x \qquad (8\text{-}1)$$

where S_{g_0} = ground roll in zero wind

S_g = ground roll with wind

w = head wind

V_T = true test ground speed at unstick, with headwind

The exponent is usually taken to be 1.85 or 1.9. This formula was derived empirically many years ago, so it seemed desirable to re-examine it. The above ratio has therefore been computed over a range of w/V_T assuming

(a) Acceleration decreasing linearly with airspeed to:

(1) 80% of its initial value

(2) 40% of its initial value

(b) Acceleration decreasing linearly with airspeed squared to:

(1) 80% of its initial value

(2) 40% of its initial value

The figures of 80% and 40% are representative of jet and propeller airplanes respectively. The ratio so computed is plotted in Figure 4, together with that given by the empirical equation. The curves for assumption (a) and (b) were indistinguishable over the range shown.

It will be seen that the cases differ little up to very high wind speeds, and also that the empirical formula agrees excellently. As its form is very convenient it is, therefore, proposed to retain it.

8.2 Correction of Air Phase for Wind: The test air distance is presently corrected for wind by adding the drift, i.e., the product of the headwind and the time in the air phase. This correction is exact (apart from wind gradients) and will be retained.

8.3 Correction for Runway Slope: An uphill slope $\sin\phi$ will decrease the excess thrust available for accelerating the airplane by $W \sin\phi$.

Following the method of section 2.2, working with a mean excess thrust, we have equation (2-2).

$$\text{Test mean excess thrust} = \frac{W_t V_{T_t}^2}{2g\, S_{g_t}} \qquad (8\text{-}2)$$

Hence, on a level runway at the test weight, air temperature and air pressure

$$\text{Mean excess thrust} = \frac{W_t V^2_{T_t}}{2g\, S_{g_t}} + W_t \sin\phi \qquad (8\text{-}3)$$

The corresponding ground roll, to reach the take-off speed V_{T_t}, will therefore be:

$$\frac{W_t V_{T_t}^2}{2g S_{g_t}} \Big/ \left\{ \frac{W_t V_{T_t}^2}{2g S_{g_t}} + W_t \sin\phi \right\}$$

$$= S_{g_t} \left\{ 1 + \frac{2g S_{g_t}}{V_{T_t}^2} \sin\phi \right\}^{-1} \qquad (8\text{-}4)$$

Thus, correction for runway slope is made by dividing the test distance by

$(1 + \frac{2gS_{g_t}}{V_{T_t}^2} \sin\phi)$, where $\sin\phi$ is the uphill slope.

8.4 Correction to Constant C_L: If the lift coefficients at take-off and at 50 feet are widely scattered, it may occasionally be desirable to correct the corresponding distances to selected values of the lift coefficients.

Ground Roll: The basic equation for the ground roll is equation (2-3)

$$S_g = \frac{W}{2g} \frac{V_T^2}{\overline{F} - \overline{D}}$$

$$= \frac{W^2}{2g} \frac{V_T^2}{W} \frac{1}{\overline{F} - \overline{D}} \tag{3-5}$$

As the lift coefficient at take-off is proportional to W/V_T^2 correction is made primarily by multiplying the test value of Sg, after correction for wind and runway slope, by the ratio

$$\left(\frac{V_T^2}{W}\right)_s \Big/ \left(\frac{V_T^2}{W}\right)_t .$$

With jet airplanes, the mean thrust will be sensibly unaltered. With propeller airplanes, however, the change of speed will also alter the thrust. From equation (A-4-7) of Appendix IV we have

$$\frac{\Delta \overline{F}}{\overline{F}_t} = \left(\frac{J}{\eta} \frac{\partial \eta}{\partial J} - 1\right)$$

But we are proposing to assume, section 7.4, that

$$A_W = -0.2$$

where from Appendix IV equation (A-4-10)

$$A_W = \frac{1}{2}\left(\frac{J}{\eta} \frac{\partial \eta}{\partial J} - 1\right)$$

i.e. $\frac{J}{\eta} \frac{\partial \eta}{\partial J} - 1 = -0.4$

The change in S_g which results from the change in thrust is given in equation (4-3) by

$$\frac{\Delta_F S_g}{S_{g_t}} = -\left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right) \frac{\Delta \overline{F}}{\overline{F}_t}$$

i. e. with the assumption of section 5 that

$$\frac{\overline{D}}{\overline{F} - \overline{D}} = 0.3$$

we have

$$\frac{\Delta_F S_g}{S_{g_t}} = -1.3\,(-0.4)\,\frac{\Delta V_T}{V_{T_t}}$$

$$= 0.52\,\frac{\Delta V_T}{V_{T_t}}$$

$$= 0.3\,\Delta\left(\frac{V_T^2}{W}\right)\Big/\left(\frac{V_T^2}{W}\right)_t \qquad \text{approximately}$$

as $\bar{V}$ is proportional to V_T.

The complete correction of the ground roll to constant take-off lift coefficient is therefore given for propeller airplanes by

$$\frac{S_{g_s}}{S_{g_t}} = \left\{\left(\frac{V_T^2}{W}\right)_s \Big/ \left(\frac{V_T^2}{W}\right)_t\right\}\left\{1 + 0.3\,\frac{\Delta\left(\frac{V_T^2}{W}\right)}{\left(\frac{V_T^2}{W}\right)_t}\right\}$$

$$= 1 + 1.3\,\frac{\Delta\left(\frac{V_T^2}{W}\right)}{\left(\frac{V_T^2}{W}\right)_t} \qquad \text{approximately} \quad (8\text{-}6)$$

and for jet airplanes by

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{V_T^2}{W}\right)_s \Big/ \left(\frac{V_T^2}{W}\right)_t \qquad (8\text{-}7)$$

For mixed propulsive systems which include propellers $\frac{\Delta \bar{F}}{\bar{F}_t}$ must be evaluated separately. Then

$$\frac{S_{g_s}}{S_{g_t}} = 1 + \frac{\Delta\left(\frac{V_T^2}{W}\right)}{\left(\frac{V_T^2}{W}\right)_t} - 1.3\,\frac{\Delta \bar{F}}{\bar{F}_t}$$

<u>Air Distance:</u> The basic equation for the air distance is equation (2-8)

$$S_a = \frac{W\,(50 + h_v)}{\bar{F} - \bar{D}}$$

The primary correction to constant lift coefficients is made by allowing for the change in h_v. We have

$$h_v = \frac{V_{5C}^2 - V_T^2}{2g} \qquad \text{in consistent units}$$

The most straightforward attack is to compute h_{v_t} and h_{v_s} using the relations

$$V_{50_s}^2 = V_{50_t}^2 \left\{ \left(\frac{V_{50}^2}{W}\right)_s \Big/ \left(\frac{V_{50}^2}{W}\right)_t \right\}$$

$$V_{T_s}^2 = V_{T_t}^2 \left\{ \left(\frac{V_T^2}{W}\right)_s \Big/ \left(\frac{V_T^2}{W}\right)_t \right\}$$

Then for jet airplanes no thrust correction is required and we have

$$\frac{S_{a_s}}{S_{a_t}} = \frac{50 + h_{V_s}}{50 + h_{V_t}} \tag{8-8}$$

With propeller airplanes, the thrust must be corrected, as for the ground roll. By a similar analysis, we have, since we are taking as mean speed, the speed V_{50} at 50 feet (Section 2.2).

$$\frac{S_{a_s}}{S_{a_t}} = \frac{50 + h_{V_s}}{50 + h_{V_t}} \left\{ 1 + 0.3 \frac{\Delta\left(\frac{V_{50}^2}{W}\right)}{\left(\frac{V_{50}^2}{W}\right)_t} \right\} \tag{8-9}$$

Again, for mixed propulsive systems using propellers $\frac{\Delta \bar{F}}{F_t}$ must be evaluated separately and substituted in the equation

$$\frac{S_{a_s}}{S_{a_t}} = \frac{50 + h_{V_s}}{50 + h_{V_t}} \left\{ 1 + \frac{\Delta\left(\frac{V_T^2}{W}\right)}{\left(\frac{V_T^2}{W}\right)_t} - 1.6 \frac{\Delta \bar{F}}{\bar{F}_t} \right\} \tag{8-10}$$

Where the factor 1.6 is the value proposed in section 5 for $\left(1 + \frac{\bar{D}}{\bar{F} - \bar{D}}\right)$

Part Time JATO: Correction of take-offs in which ATO is used for part time, which is examined in the next section, is inherently complicated by the fixed endurance characteristic of the usual rockets. As correction to constant lift coefficient is rarely needed anyhow, it is not considered desirable to treat this case here. If desired, any particular case can be dealt with by an adaptation of the above analysis and that of the next section.

9. Part-Time Assistance to Take-Off:

As has already been remarked, rocket assistance over the whole of the take-off is no different in principle from any other method of providing the required thrust. Many ATO units are, however, of limited endurance and are operated only over the last part of the take-off, when their assistance is most helpful. It is this "part-time" feature of ATO which necessitates a separate treatment.

To make use of earlier analysis it is desirable to convert the actual thrust F_R of the ATO into an effective mean thrust $\bar{F}_R$. It is shown in Appendix V that this may be approximated satisfactorily by means of the equation

$$\overline{F}_R = \frac{S_R}{S} F_R \qquad (9\text{-}1)$$

where

$S_R =$ distance covered with ATO operating

$S =$ total distance covered during the phase

Consideration will be given to standardization to two cases only:

a. ATO stops as airplane passes 50 foot screen

b. ATO stops as airplane takes off

(a) will be the more usual case, but (b) may be needed for the case of a short runway with no obstructions ahead of it.

Consider first the air phase. The ATO units will probably be fired too early or too late to burn out at exactly the desired point, so correction must be made. The test mean thrust is deduced by inserting test values of F_R, S_R, and S in equation (9-1) above. The standard mean thrust is also easily deduced, as the ratio S_R/S will be equal to unity in case (a), and zero in case (b). Standardization is then effected by substituting these mean thrusts in equation (4-5).

Now consider the ground phase. Again, the test mean thrust is readily evaluated using equation (9-1). The point at which the ATO is fired under standard conditions, must, however, be adjusted to make it burn out at 50 feet or at unstick, as desired. To do this, we first estimate the firing time under standard conditions during the air phase. In case (b) this is, of course, zero. In case (a) we may approximate it by the equation

$$t_{R_{a_s}} = 2\, S_{a_s}/(V_{T_s} + V_{50_s}) \qquad (9\text{-}2)$$

when $t_{R_{a_s}} =$ JATO time in air phase under standard conditions.

$S_{a_s} =$ standard air distance

$V_{T_s} =$ standard take-off speed

$V_{50_s} =$ standard speed at 50 feet

If now, the time for which the ATO was operating in the air phase during the test was $t_{R_{a_t}}$, we must correct the test time of ATO operation in the ground phase by adding to it

$$t_R = t_{R_{a_t}} - t_{R_{a_s}} \qquad (9\text{-}3)$$

(The thrust and endurance of the ATO units may be assumed equal under test and standard conditions.) Allowing for this, it is shown in Appendix V that equation

(4-3) may be adapted to give the form

$$\begin{aligned}\frac{\Delta S_g}{S_{g_t}} = & \left\{2.3 + \frac{2.3}{\mathcal{R} - 1.3}\right\}\frac{\Delta W}{W_t} \\ & - \left\{1 + \frac{0.7}{\mathcal{R} - 1.3}\right\}\frac{\Delta\sigma}{\sigma_t} \\ & - \frac{1.3}{\mathcal{R} - 1.3}\;\frac{\Delta t_{R_g}}{t_{R_{g_t}}} \\ & - \left\{1.3 + \frac{1.7}{\mathcal{R} - 1.3}\right\}\frac{\Delta \bar{F}_b}{\bar{F}_t}\end{aligned} \qquad (9\text{-}4)$$

where $\mathcal{R} = F/F_R$

$\bar{F}$ = total mean effective thrust

$\bar{F}_R$ = effective mean test ATO thrust

$\Delta\bar{F}_b$ = correction to mean thrust of basic propulsion systems

$t_{R_{g_t}}$ = test time of ATO operation during ground phase

This correction process is quite a little more complicated than that for take-off without part time assistance, but the complication is not too formidable. It would seem inevitable at the present time.

10. Combined Formulae for Propeller Airplanes:

For airplanes using propeller thrust only for take-off it may be convenient to substitute from the proposed generalized thrust equations (7-2) or (7-3) and (7-5) into equations (4-1) and (4-5).

Recapitulating and inserting the proposed values of h_v, $\bar{D}/(\bar{F} - \bar{D})$ and so on, we have

$$\frac{\Delta S_g}{S_{g_t}} = 2.3\,\frac{\Delta W}{W_t} - \frac{\Delta\sigma}{\sigma_t} - 1.3\,\frac{\Delta\bar{F}}{\bar{F}_t} \qquad (10\text{-}1)$$

$$\frac{\Delta S_a}{S_{a_t}} = 2.3\,\frac{\Delta W}{W_t} - 0.7\,\frac{\Delta\sigma}{\sigma_t} - 1.6\,\frac{\Delta\bar{F}}{\bar{F}_t} \qquad (10\text{-}2)$$

for all except light airplanes.

$$= 2.0\,\frac{\Delta W}{W_t} - 0.4\,\frac{\Delta\sigma}{\sigma_t} - 1.6\,\frac{\Delta\bar{F}}{\bar{F}_t} \qquad (10\text{-}3)$$

for light airplanes.

Also, for fixed pitch propellers

$$\frac{\Delta\bar{F}}{\bar{F}_t} = 1.1\,\frac{\Delta\sigma}{\sigma_t} - 0.1\,\frac{\Delta W}{W_t} \qquad (10\text{-}4)$$

at constant engine speed

$$\frac{\Delta \bar{F}}{\bar{F}_t} = 1.1\frac{\Delta\sigma}{\sigma_t} + 0.4\frac{\Delta T_a}{T_{a_t}} - 0.1\frac{\Delta W}{W_t} \qquad \text{at full throttle} \tag{10-5}$$

while for constant speed propellers

$$\frac{\Delta \bar{F}}{\bar{F}_t} = 0.7\frac{\Delta P}{P_t} + 0.5\frac{\Delta\sigma}{\sigma_t} - 0.5\frac{\Delta N}{N_t} - 0.2\frac{\Delta W}{W_t} \tag{10-6}$$

Substituting, as appropriate, we have:

<u>Light airplanes with fixed pitch propellers</u>

At constant engine speed

$$\frac{\Delta S_g}{S_{g_t}} = 2.4\frac{\Delta W}{W_t} - 2.4\frac{\Delta\sigma}{\sigma_t} \tag{10-7}$$

and

$$\frac{\Delta S_a}{S_{a_t}} = 2.2\frac{\Delta W}{W_t} - 2.2\frac{\Delta\sigma}{\sigma_t} \tag{10-8}$$

at full throttle

$$\frac{\Delta S_g}{S_{g_t}} = 2.4\frac{\Delta W}{W_t} - 2.4\frac{\Delta\sigma}{\sigma_t} + 0.5\frac{\Delta T_a}{T_{a_t}} \tag{10-9}$$

$$\frac{\Delta S_a}{S_{g_t}} = 2.2\frac{\Delta W}{W_t} - 2.2\frac{\Delta\sigma}{\sigma_t} + 0.6\frac{\Delta T_a}{T_{a_t}} \tag{10-10}$$

<u>Light airplanes with constant speed propellers</u>

$$\frac{\Delta S_g}{S_{g_t}} = 2.6\frac{\Delta W}{W_t} - 1.7\frac{\Delta\sigma}{\sigma_t} - 0.7\frac{\Delta N}{N_t} - 0.4\frac{\Delta P}{P_t} \tag{10-11}$$

$$\frac{\Delta S_a}{S_{a_t}} = 2.3\frac{\Delta W}{W_t} - 1.2\frac{\Delta\sigma}{\sigma_t} - 0.8\frac{\Delta N}{N_t} - 1.1\frac{\Delta P}{P_t} \tag{10-12}$$

<u>Heavy airplanes with constant speed propellers</u>

$$\frac{\Delta S_g}{S_{g_t}} = 2.6\frac{\Delta W}{W_t} - 1.7\frac{\Delta\sigma}{\sigma_t} - 0.7\frac{\Delta N}{N_t} - 0.1\frac{\Delta P}{P_t} \tag{10-13}$$

$$\frac{\Delta S_a}{S_{a_t}} = 2.6\frac{\Delta W}{W_t} - 1.5\frac{\Delta\sigma}{\sigma_t} - 0.8\frac{\Delta N}{N_t} - 1.1\frac{\Delta P}{P_t} \tag{10-14}$$

<u>Large Corrections:</u>

If rather large corrections are involved ($|\Delta S| > 0.2\ S_t$) it may be

preferable, again, to use the exponential form similar to equations (4-3a) and (4-5a). For example, instead of equation (10-13) one would use the alternative equation

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2.6}\left(\frac{\sigma_s}{\sigma_t}\right)^{-1.7}\left(\frac{N_s}{N_t}\right)^{-0.7}\left(\frac{P_s}{P_t}\right)^{-0.9} \tag{10-13a}$$

11. Non-Dimensional Methods:

"Non-dimensional" methods of performance reduction are in general use for reducing level speed performance tests on turbojet airplanes to standard conditions. Consideration of their use for take-off reduction is in hand as a separate project, but a brief general discussion would be of value here.

Making assumptions similar to those used above, of constant lift coefficient at unstick and at 50 feet it can be shown that

$$\frac{gS}{\theta} = f_1\left(\frac{\overline{F}}{\delta}, \frac{W}{\delta}\right) \tag{11-1}$$

where

- S = distance to unstick or to clear a screen of height $(50/\theta)$ feet.
- $\overline{F}$ = total net thrust
- W = airplane gross weight
- θ = (air temperature $^{\circ}$K)/288
- δ = air pressure, atmospheres

With the simple turbojet airplane we can substitute for $\overline{F}/\delta$ in terms of $N/\sqrt{\theta}$, if desired, and write

$$\frac{gS}{\theta} = f_2\left(\frac{N}{\sqrt{\theta}}, \frac{W}{\delta}\right) \tag{11-2}$$

Thus, if the ground roll and distance to $(50/\theta)$ feet were measured at the standard values of $(N/\sqrt{\theta})$ and (W/δ) the standard values of ground roll and distance to 50 feet could be very readily deduced. Alternatively, these standard values could be deduced by interpolation from tests made over suitable ranges of N and W.

With propeller airplanes also, substitution for $\overline{F}/\delta$ is permissible since with a propeller at given Mach number

$$\overline{F}/\delta = f_3\left(\frac{N}{\sqrt{\theta}}, \frac{Q}{\delta}\right) \tag{11-3}$$

The mean Mach number of each phase is unchanged by test conditions if the lift coefficients and W/δ are held constant, so we may substitute from (11-3) into (11-1) and write

$$\frac{gS}{\theta} = f_4\left(\frac{N}{\sqrt{\theta}}, \frac{Q}{\delta}, \frac{W}{\delta}\right) \tag{11-4}$$

Hence, the standard tests could be deduced directly from tests at the desired values of the above three variables, or by interpolation. To be able to do this one must be able to vary Q/δ independently of $N/\sqrt{\theta}$. This can be done with the usual piston engine installation with constant speed propeller but will be impossible with some turbo-propeller installations with single coordinated engine controls.

It should be noted that in the above it is assumed that the lift coefficient at height $50/\theta$ feet is unchanged in test conditions, whereas in the earlier treatment in this Report the lift coefficient at height of 50 feet is assumed constant. However, these assumptions will usually be identical.

It is not proposed to prejudice conclusions of another project by extended discussion here, but certain preliminary conclusions are readily drawn. Firstly, the method is very attractive if it is in fact practicable to make tests at the desired standard values of the independent variables, as the reduction process then amounts only to dividing the test ground roll and distance to $(50/\theta)$ feet by the test value of θ. Secondly, if precise control of any one of the variables to the desired standard values is not practicable, tests must be made over a range of that variable to permit interpolation. The poor repeatability of take-off tests may make it uneconomical to do this, at least if tests are to be made over a range of two or more variables, unless final data are also required over a range of variables, for example, of weight, pressure, altitude, and air temperature. When tests and reduced data are both required over a wide range of conditions, however, the technique again appears very attractive.

12. Verification of Formulae:

The reduction formulae have been checked:

a. Against tests over a wide range of pressure and weight, on a light airplane with fixed pitch propeller.

b. Against tests over a wide range of weight on a medium propeller driven airplane.

c. Against tests over a wide range of weight and a range of power and air temperature on a heavy propeller bomber.

d. Against tests over a very wide range of air temperature, air pressure, and a moderate range of weight on two jet fighters.

e. Against design firm's predictions over a wide range of weight, air temperature and air pressure for a medium jet bomber.

Light Fixed Pitch Airplane: Take-offs were measured on this airplane from a runway at Edwards and from sod near sea level. The test air temperatures were very close to standard, so the data give no check of that correction. They do, however, indicate the suitability of the pressure and weight corrections.

Corrections for pressure were made using formulae produced for the particular airplane before the generalized equations (10-7) thru (10-10) were available.

The relations assumed were for 20° flap,

$$\frac{\Delta S_g}{S_{g_t}} = 2.5 \frac{\Delta P_a}{P_{a_t}}$$

and

$$\frac{\Delta S_a}{S_{a_t}} = 2.0 \frac{\Delta P_a}{P_{a_t}}$$

These factors, 2.5, and 2.0, compare with 2.4 and 2.2 in the general equations. The differences are not significant, as they correspond to about 1% of the ground roll (3 ft) and 1-1/2% of the air distance (3 ft). The distance to 50 feet so corrected, is plotted against gross weight in Figure 5 for concrete and sod runway. It will be seen that the distance to reach 50 feet from the concrete runway is, after correction to sea level, about 20-30 feet less than that from the sod runway. This difference, about 10% of the ground roll, is roughly what would be expected from the difference in runway surface. Thus, the data indicate that the proposed reduction formulae are not seriously in error as far as pressure corrections are concerned.

To check the weight correction, a plot has also been made in Figure 5 of $\log_{10} S_{50}$ against $\log_{10} W$. The proposed relations for ground and air distances would correspond to a slope of 2.3 for the total distance to 50 feet. This also is plotted in the Figure 5. It will be seen that the proposed slope agrees satisfactorily, particularly as test weights on such airplanes differ little from standard, and hence, no great degree of precision is required of the correction.

Medium Propeller Airplane: The tests on this airplane covered a wide range of weight, 100,000 to 160,000 lbs., but insufficient range of other parameters to give a satisfactory check of the reduction formulae. The test data have been corrected to a selected power and to standard air temperate and air pressure. The corrected distance is plotted against gross weight in Figures 6A and 6B, again on a logarithmic scale. To reduce scatter, the ground rolls have been corrected to a mean lift coefficient. The slopes given by the generalized equations are drawn in; it will be seen that the proposed slopes agree with the test data within the errors of the data.

Heavy Propeller Airplane: Tests on the Model A and Model B of this type each covered a wide range of weight. Taken together, (the airplanes being very similar) they also covered a considerable range of air temperature and power.

The ground rolls and the air distances have been corrected to constant lift coefficients at take-off and at 50 feet, to 15°C. sea level and 3000BHP/engine. They are plotted against weight using a logarithmic scale in Figure 7A. The slope corresponding to the proposed correction has been drawn in. It will be seen that the proposed slope fits the experimental data very well. There is no difference apparent between the two models. The distances have been further corrected for weight, using the proposed correction, and are plotted against power and air density in Figure 7B. It will be seen that the ground roll corrections appear to bring the test data together well within the experimental scatter. The evidence for the air phase is less satisfactory, but does not disprove the proposed corrections.

Jet Fighter No. 1: The tests on this airplane were designed to investigate the effects of air temperature and pressure, and airplane gross weight on take-off performance. They covered a range of 6000 ft in pressure altitude, 20°C in air temperature, and 20% in gross weight.

From static thrust measurements it appeared that over the take-off range of conditions

$$\frac{\Delta \bar{F}}{\bar{F}_t} = \frac{\Delta P_a}{P_{a_t}} - 1.3\frac{\Delta T_a}{T_{a_t}} \tag{12-1}$$

$$= \frac{\Delta \delta}{\delta_t} - 1.3\frac{\Delta T_a}{T_{a_t}} \tag{12-2}$$

where $\delta = P_{a_t}$/Standard sea level air pressure

But from equations (10-1) and (10-2)

$$\frac{\Delta S_g}{S_{g_t}} = 2.3\frac{\Delta W}{W_t} - \frac{\Delta \sigma}{\sigma_t} - 1.3\frac{\Delta \bar{F}}{\bar{F}_t} \tag{12-3}$$

and

$$\frac{\Delta S_a}{S_{a_t}} = 2.3\frac{\Delta W}{W_t} - 0.7\frac{\Delta \sigma}{\sigma_t} - 1.6\frac{\Delta \bar{F}}{\bar{F}_t} \tag{12-4}$$

also, $$\sigma = \delta \Big/ \left(\frac{T_a}{288}\right)$$

hence $$d\sigma = \frac{d\delta}{\left(\frac{T_a}{288}\right)} - \frac{\delta}{\left(\frac{T_a}{288}\right)}\frac{dT_a}{T_a}$$

i.e., $$\frac{d\sigma}{\sigma} = \frac{d\delta}{\delta} - \frac{dT_a}{T_a}$$

and, hence, approximately,

$$\frac{\Delta \sigma}{\sigma_t} = \frac{\Delta \delta}{\delta_t} - \frac{\Delta T_a}{T_{a_t}} \tag{12-5}$$

Substituting from (12-2) and (12-5) into (12-3) and regrouping the terms, we have

$$\frac{\Delta S_g}{S_{g_t}} = 2.3\frac{\Delta W}{W_t} - 2.3\frac{\Delta \delta}{\delta_t} + 2.7\frac{\Delta T_a}{T_{a_t}} \tag{12-6}$$

Similarly, for the air distance

$$\frac{\Delta S_a}{S_{a_t}} = 2.3\frac{\Delta W}{W_t} - 2.3\frac{\Delta \delta}{\delta_t} + 2.8\frac{T_a}{T_{a_t}} \tag{12-7}$$

If we multiply each side of (12-6) and (12-7) by S_{g_t} and S_{a_t} respectively, we have, for the total distance of 50 feet $S_{50} = S_g + S_a$, approximating

$$\Delta S_{50} = \Delta S_g + \Delta S_a$$

$$= 2.3\, S_{50_t}\left(\frac{\Delta W}{W_t} - \frac{\Delta \delta}{\delta_t}\right) + 2.7\, S_{50_t}\, \frac{\Delta Ta}{T_{a_t}} \quad \text{approximately}$$

$$\frac{\Delta S_{50}}{S_{50_t}} = 2.3\left(\frac{\Delta W}{W_t} - \frac{\Delta \delta}{\delta_t}\right) + 2.7\, \frac{\Delta Ta}{T_{a_t}} \qquad (12\text{-}8)$$

Integrating equations (12-6) and (12-8) we have,

$$\log S_{g_t} = 2.3 \log\left(\frac{W_t}{\delta_t}\right) + 2.7 \log T_{a_t} + \text{constant} \qquad (12\text{-}9)$$

$$= 2.3 \log\left(\frac{W_t}{\delta_t}\right) + 2.7 \log T_{a_t} + \text{constant} \qquad (12\text{-}10)$$

In figures 8A and 8B log S_{g_t} and log S_{50_t} are plotted against log T_{a_t} for each combination of W and δ, and lines are drawn through the experimental points with a slope of 2.7. It will then be seen that these lines agree well with the experimental points.

Accepting this relation between distance and air temperature the test distances have been corrected for temperature. The logarithm of the distances so corrected are plotted against log W/δ in Figure 8C. It will be seen these points also agree well with the predicted slopes.

Thus, the tests on this airplane agree very well with the proposed reduction equations.

<u>Jet Fighter No. 2:</u> The tests on this airplane were designed, as were those on Jet Fighter No. 1, to determine the take-off performance over a wide range of air temperature and pressure and a moderate range of weight.

The equations for the ground roll and air distance were deduced as for Jet Fighter No. 1. They were

$$\log S_{g_t} = 2.3 \log \frac{W_t}{\delta_t} + 3.26 \log T_{a_t} + \text{constant} \qquad (12\text{-}11)$$

and

$$\log S_{a_t} = 2.3 \log \frac{W_t}{\delta_t} + 3.48 \log T_{a_t} + \text{constant} \qquad (12\text{-}12)$$

As the air distance is only about 20% of the total, we may deduce as the relation for the total distance to 50 feet

$$\log S_{50_t} = 2.3 \log \frac{W_t}{\delta_t} + 3.3 \log T_{a_t} + \text{constant} \qquad (12\text{-}13)$$

As with the previous example, $\log_{10} S_{g_t}$ and $\log_{10} S_{50_t}$ have first been plotted

against $\log_{10} T_{a_t}$ (^{o}K) for the experimental data for each value of W and δ to check the proposed temperature correction (Figures 9A and 9B), and curves with the predicted slopes have been drawn through the groups of points. It will be seen that the proposed correction is satisfactory, although the data suggests that it may be a little too large.

The values of $\log_{10} S_{g_t}$ and $\log_{10} S_{50_t}$ given by these curves for 22°C ($\log T_{a_t} = 2.47$) have been cross-plotted against $\log_{10} (W/\delta)$ in Figure 9C. Here it will be seen that the data for the two gross weights fit together well, but that the experimental slope is no greater than 2.0 and markedly less than the predicted slope of 2.3. So low a slope for ground roll is hard to explain if it is accepted that all take-offs were made at indicated speeds 130 mph and 120 mph at the high and low gross weights respectively, as a slope of 2.0 would only be expected if there were no drag. However, in the absence of measured take-off speeds, further investigation is not possible. This apparent error of 15% in prediction would lead to an error in the corrected distances of 1% for every 1000 feet difference between test and standard altitudes. This would appear tolerable. Later tests on another model, not reproduced here because the results were much more scattered, also tended to support a slope against log W/δ of 2.0 or less.

Medium Jet Bomber: This case has been approached differently, using the Operating Manual. Accepting the figures given by the performance charts for the ground roll under one set of conditions, the proposed reduction formulae have been used to compute the performance under widely different conditions. This computed performance has then been compared with that given by the charts. The results were as follows:

a. Correcting from 61% of maximum gross weight, Sea Level, 60°F to maximum gross weight, 6000 ft., 38°F :-
computed distance = 0.96 x chart distance

b. Corrections from 83% W_{max} 3000 ft., 120°F to 83% W_{max}, 3000 ft., 0°F :-
computed distance = 1.01 x chart distance

c. Corrections from 83% W_{max} Sea Level, 60°F to 83% W_{max}, 6000 ft., 60°F :-
computed distance = 0.97 x chart distance

Here also, the agreement is fairly good.

12. Acknowledgements:

Equations (4-1) and (4-4) were originated by Captain Paul E. Shoemacher. The theoretical treatment of the speed increase on the climb to 50 feet and the empirical modification of the results, discussed in paragraph 3, and illustrated in Figures 1A and 2, were the work of P. A. Hufton, then of the Royal Aircraft Establishment, Farnborough, England.

DISCUSSION:

The proposed reduction formulae are summarized in Appendix VII.

The basic formulae can be used, with suitable numerical constants, for any propulsive system, including mixed systems, and systems using part time assistance or boost. The formulae should be easy to apply, with the possible exception of that for part-time assistance; even then, the complication is not great.

For turbo-jet airplanes, the formulae check satisfactorily with experimental data over wide ranges of all parameters. For one airplane, the experimental rate of variation of take-off distance with weight and air pressure was 15% less than that predicted, corresponding to an error of 1% in corrected distance for every 1000 feet difference between test and standard pressure altitudes. However, this error is considered tolerable. Also, the experimental value is open to attack on theoretical grounds and was not supported by measurement of take-off speeds.

For fixed pitch propeller airplanes, the corrections for weight and air pressure check satisfactorily with experiment. No data were available by which to check the temperature corrections.

For airplanes with constant speed propellers, the weight corrections check well with experiment over a wide range of test weight. The power and density corrections could not be fully checked with the data available, but they were shown to at least approximate the correct values.

It should be noted that these formulae should not be used to correct for big differences between test and standard conditions if the take-off acceleration is very low (for example if $(V_T^2/2g\ S_g)$ is less than 0.1 in consistent units). For such cases any general method of standardization other than one based on interpolation between test data is liable to be inaccurate, and care should be taken either to make tests under near standard conditions or to cover a large enough range of test conditions to permit reliable interpolation or extrapolation.

The methods assume that the lift coefficients under test and standard conditions are the same both at take-off and at 50 feet. This assumption has been made on general grounds rather than from empirical analysis.

The assumed propeller characteristics should be checked from time to time, particularly if unusual installations are to be tested. No method has been proposed by which to estimate the thrust corrections for propellers suffering marked compressibility losses as reliable generalized data were not to hand at the time of writing. The propeller tip Mach number should be checked if it is expected to be high (for example, with an ungeared propeller of large diameter) and special consideration given if it exceeds unity.

From the brief consideration given them, it appears that, in some cases at least, non-dimensional methods may be an attractive alternative to the above methods, requiring no numerical assumptions about thrust changes whether of a propeller or a turbo jet engine. This type of method is being further considered as a separate project.

CONCLUSIONS AND RECOMMENDATIONS:

Correction formulae have been derived which are easy to use and apply, with

suitable numerical constants, to airplanes with any type of propulsive system.

Generalized numerical constants are proposed for insertion in these formulae. The experimental data available supports these constants but is insufficient to check them completely. However, the propeller assumptions should be checked occasionally.

Non-dimensional methods, which show promise of being a convenient alternative in some cases at least, are being investigated as a separate project.

It is recommended that the proposed methods be used for standardization of future take-off test data.

REFERENCES

1. The Elements of Aerofoil and Airscrew Theory. H. Glauert. Cambridge University Press, England, 1948.

APPENDIX I

Approximation of the Mean Excess Thrust

INTRODUCTION:

The excess thrust and hence the acceleration decrease with increase of airspeed until at take-off, they are about 80% (for jet airplanes) or 40% (for propeller airplanes) of their initial values. The slope of the curve can usually be approximated closely by assuming it to be linear with regard to either speed or speed squared. Representative cases are shown in Figure 10. We will show that in either case, a figure of 0.75 V_T for $\bar{V}$ will give an acceleration very close to the true value over the required range of slopes. This being so, this value for $\bar{V}$ should be satisfactory.

NOTATION:

Symbol	Definition
D	total resistance
$\bar{D}$	value of D when $V = \bar{V}$
F	total net thrust
$\bar{F}$	value of F when $V = \bar{V}$
r_1, r_2	constants in acceleration - speed relations
S	distance to attain speed V
S_g	distance to unstick
t	time
V	true speed
V_T	take-off speed
v	V/V_T
$\bar{V}$	value of V at which $F - D = WV_T^2/2gS_g$ (paragraph 2 of main text)
α	acceleration
α_o	acceleration at zero forward speed
$\bar{\alpha}$	mean acceleration $= (V_T^2/2\ S_g)$

<u>Excess Thrust Linear With Regard to Speed</u>: We will work in terms of the acceleration α, where

$$\alpha = \frac{g}{W}(F - D) \qquad (A1\text{-}1)$$

We will write

$$\alpha = \alpha_o (1 - r_1 v) \qquad (A1\text{-}2)$$

Then we have $$S_g = \int_o^{S_g} dS$$

$$= \int_o^{V_T} \left\{ \frac{dS}{dt} \Big/ \frac{dV}{dt} \right\} dV$$

$$= \int_o^{V_T} \frac{VdV}{\alpha}$$

$$= \frac{V_T^{\,2}}{\alpha_o} \int_o^1 \frac{vdv}{1 - r_1 v}$$

$$= -\frac{V_T^{\,2}}{r_1^{\,2}\alpha_o} \int_o^1 \left\{ d(r_1 v) + \frac{d(-r_1 v)}{1 + (-r_1 v)} \right\}$$

$$= -\frac{V_T^{\,2}}{r_1^{\,2}\alpha_o} \left\{ r_1 + \log_e (1 - r_1) \right\} \qquad (A1\text{-}3)$$

Let us denote the mean acceleration , which would produce the take-off speed V_T in the ground roll distance S_g, by $\bar{\alpha}$. Then

$$\bar{\alpha} = \frac{V_T^2}{2S_g} \tag{A1-4}$$

But from equation (A1-3)

$$\frac{V_T^2}{2S_g} = - \frac{\frac{1}{2}\alpha_o r_1^2}{r_1 + \log_e (1 - r_1)} \tag{A1-5}$$

Hence

$$\bar{\alpha} = - \frac{\frac{1}{2}\alpha_o r_1^2}{r_1 + \log_e (1 - r_1)} \tag{A1-6}$$

If an assumption $\bar{V} = 0.75\, V_T$ is to be satisfactory, $\bar{\alpha}$ must be sensibly equal to the acceleration $\alpha_{0.75}$ at that speed. The ratio $\alpha_{0.75}/\bar{\alpha}$ has been computed over a range of r_1, with the results tabulated below.

r_1	0.2	0.4	0.6	0.8
$\alpha_{0.75}/\bar{\alpha}$	0.98	0.97	0.97	1.03

It will be seen that the above assumption is satisfactory over the likely range of r_1. The approximation gives a good estimate of $\bar{\alpha}$ and hence of $(\bar{F} - \bar{D})$. Also, as the ratio $\alpha_{0.75}/\bar{\alpha}$ varies very slowly with r_1 it will also give a good estimate of the change in $(\bar{F} - \bar{D})$ between test and standard conditions, even if the test and standard values of r_1 may differ appreciably.

<u>Acceleration Linear with Regard to Square of Speed</u>: In this case, we will write

$$\alpha = \alpha_o (1 - r_2 v^2) \tag{A1-7}$$

Then as before

$$S_g = \int_0^{V_T} \frac{VdV}{\alpha}$$

$$= \frac{V_T^2}{\alpha_o} \int_0^1 \frac{v\,dv}{1 - r_2 v^2}$$

$$= \frac{V_T^2}{2\alpha_o r_2} \int_0^1 \frac{d(r_2 v^2)}{1 - r_2 v^2}$$

$$= \frac{V_T^2}{2\alpha_o r_2} \Big[- \log_e (1 - r_2 v^2) \Big]_0^1$$

$$= - \frac{V_T^2}{2\alpha_o r_2} \log_e (1 - r_2) \qquad \text{(A1-8)}$$

But we also have, as before, (equation (A1-4),

$$\bar{\alpha} = \frac{V_T^2}{2S_g}$$

Hence $$\bar{\alpha} = - \alpha_o \frac{r_2}{\log_e (1 - r_2)} . \qquad \text{(A1-9)}$$

But $$\alpha_{0.75} = \alpha_o \left\{ 1 - (0.75)^2 r_2 \right\} \qquad \text{(A1-10)}$$

Hence $$\alpha_{0.75}/\bar{\alpha} = - \frac{1}{r_2} \left\{ 1 - (0.75)^2 r_2 \right\} \log_e (1 - r_2) \qquad \text{(A1-11)}$$

$\alpha_{0.75}/\bar{\alpha}$ has been computed for this case also over the likely range of r_2, with the results tabulated below:

r_2	0.2	0.4	0.6	0.8
$\alpha_{0.75}/\bar{\alpha}$	0.99	0.99	1.01	1.11

It will be seen that over the likely range of r_2 (around 0.2 to 0.6) the ratio is again very nearly unity, and that $\alpha_{0.75}$ will approximate $\bar{\alpha}$ closely. That is, the excess thrust at 0.75 V_T is a close approximation to the mean excess thrust.

APPENDIX II

Derivation of Performance Reduction Equations

NOTATION

Symbol	Definition
C_{D_g}	drag coefficient during ground roll
C_{L_g}	lift coefficient during ground roll
$C_{L_{T.O.}}$	lift coefficient at take-off
D_a	aerodynamic drag
$\overline{D}$	total resistance at mean speed $\overline{V}$
$\overline{F}$	total net thrust at mean speed $\overline{V}$
h_v	$(V^2_{50} - V_T^{\,2}) / 2g$
K, K', K'', K'''	constants of proportionality used temporarily and defined locally
L	aerodynamic lift during ground roll
S_W	gross wing area
$\overline{V}$	mean speed during phase = $0.75\ V_T$ for ground roll = V_{50} for air phase
V_T	true speed at take-off
V_{50}	true speed at 50 ft.
W	gross weight of airplane
μ	coefficient of rolling friction of tires
σ	relative density of air = local density/standard sea level density

Subscripts 't' and 's' denote test and standard conditions respectively.

<u>Derivation of Equation (4-1)</u>: We have (equation (2-2) of main text)

$$S_g \propto \frac{W^2}{\sigma} \quad \frac{1}{\overline{F} - \overline{D}} \qquad \text{(A2-1)}$$

Hence, if subscripts 't' and 's' denote test and standard conditions respectively

$$\frac{S_{g_t}}{S_{g_s}} = \left(\frac{W_t}{W_s}\right)^2 \frac{\sigma_s}{\sigma_t} \quad \frac{\overline{F}_s - \overline{D}_s}{\overline{F}_t - \overline{D}_t}$$

$$= \frac{W_t}{W_s} \frac{\sigma_s}{\sigma_t} \left[\frac{W_t}{W_s} \frac{\overline{F}_s}{\overline{F}_t - \overline{D}_t} - \frac{W_t}{W_s} \frac{\overline{D}_s}{\overline{F}_t - \overline{D}_t} \right] \qquad \text{(A2-2)}$$

Now $\overline{D}$ is the total resistance at speed $\overline{V} = 0.75\ V_T$. We have

$$D = \mu(W - L) + D_a$$

where L = aerodynamic lift

D_a = aerodynamic drag

μ = coefficient of rolling friction of tires

Hence, if C_{D_g} and C_{L_g} are the drag and lift coefficients during the ground roll, S_W is the gross wing area,

$$\overline{D} = \mu W + \tfrac{1}{2} \rho \overline{V}^2 S_W \ (C_{D_g} - \mu C_{L_g})$$

$$= \mu W + \tfrac{1}{2} \rho V_T^2 S_W \ (0.75^2)\ (C_{D_g} - \mu C_{L_g}) \qquad \text{(A2-3)}$$

If, further, $C_{L_{T.O.}}$ is the lift coefficient at take-off

$$W = \tfrac{1}{2}\rho V_T^2 S_W C_{L_{T.O.}} \tag{A2-4}$$

Substituting for $\frac{1}{2}\rho V_T^2 S_W$ from (A2-4) into (A2-3), we have

$$\bar{D} = \mu W + (0.75)^2 W \frac{C_{D_g} - \mu C_{L_g}}{C_{L_{T.O.}}} \tag{A2-5}$$

Hence, assuming C_{D_g}, C_{L_g}, $C_{L_{T.O.}}$ and μ constant, we have

$$\frac{\bar{D}_s}{\bar{D}_t} = \frac{W_s}{W_t}$$

$$\bar{D}_t = \bar{D}_s \frac{W_s}{W_t} \tag{A2-6}$$

Substituting for $\bar{D}_s \frac{W_s}{W_t}$ from (A2-6) into (A2-2), we thus have

$$\frac{S_{g_t}}{S_{g_s}} = \frac{W_t}{W_s}\frac{\sigma_s}{\sigma_t}\left[\frac{W_t}{W_s}\frac{\bar{F}_s}{\bar{F}_t - \bar{D}_t} - \frac{\bar{D}_t}{\bar{F}_t - \bar{D}_t}\right]$$

$$= \frac{W_t}{W_s}\frac{\sigma_s}{\sigma_t}\left[\frac{W_t}{W_s}\frac{\bar{F}_s}{\bar{F}_t - \bar{D}_t} + 1 - \frac{\bar{F}_t}{\bar{F}_t - \bar{D}_t}\right] \tag{A2-7}$$

From equation (2-3) of the main text, we may deduce that

$$\bar{F}_t - \bar{D}_t = W_t \frac{V_T^2}{2g\, S_{g_t}} \tag{A2-8}$$

Hence, we may rewrite equation (A2-7) in a form more directly related to experimental data, as follows:

$$\frac{S_{g_t}}{S_{g_s}} = \frac{W_t}{W_s}\frac{\sigma_s}{\sigma_t}\left[\frac{2g\,S_{g_t}}{W_t\,V_T^{\,2}}\left(\frac{W_t}{W_s}\,\overline{F}_s - \overline{F}_t\right) + 1\right] \tag{A2-9}$$

or

$$\frac{S_{g_s}}{S_{g_t}} = \frac{\dfrac{W_s}{W_t}\dfrac{\sigma_t}{\sigma_s}}{\dfrac{2g\,S_{g_t}}{W_t V_{T_t}^{\,2}}\left(\dfrac{W_t}{W_s}\,\overline{F}_s - \overline{F}_t\right) + 1} \tag{A2-10}$$

Derivation of equation (4-4): Equation (4-4) of the main text may be derived somewhat similarily. We have (equation (2-8) of main text)

$$S_a = \frac{50\,W}{\overline{F} - \overline{D}}\cdot\frac{h_v + 50}{50} \tag{A2-11}$$

and hence,

$$\frac{S_{a_t}}{S_{a_s}} = \frac{W_t}{W_s}\,\frac{\overline{F}_s - \overline{D}_s}{\overline{F}_t - \overline{D}_t}\,\frac{h_{v_t} + 50}{h_{v_s} + 50} \tag{A2-12}$$

We have assumed that the test and standard lift coefficients are the same, both at take-off and at 50 ft. Hence

$$\frac{V_{50_t}^{\,2}}{V_{50_s}^{\,2}} = \frac{V_{T_t}^{\,2}}{V_{T_s}^{\,2}} = \frac{W_t}{W_s}\frac{\sigma_s}{\sigma_t} \tag{A2-13}$$

Hence, we have $h_{v_s} = \dfrac{V_{50_s}^{\,2} - V_{T_s}^{\,2}}{2g} = \dfrac{W_s}{W_t}\,\dfrac{\sigma_t}{\sigma_s}\,\dfrac{V_{50_t}^{\,2} - V_{T_t}^{\,2}}{2g}$

$$= \frac{W_s}{W_t}\,\frac{\sigma_t}{\sigma_s}\,h_{v_t} \tag{A2-14}$$

Also, if the drag and lift coefficients at the mean air speed $\overline{V}$ are constant

$$\frac{\overline{D}_s}{\overline{D}_t} = \frac{\frac{1}{2}\rho_s \overline{V}_s^{\,2} S_W C_D}{\frac{1}{2}\rho_t \overline{V}_t^{\,2} S_W C_D} = \frac{\frac{1}{2}\rho_s \overline{V}_s^{\,2} S_W C_L}{\frac{1}{2}\rho_t \overline{V}_t^{\,2} S_W C_L}$$

$$= \frac{\frac{1}{2}\rho_s V_{T_s}^{\,2} S_W C_L}{\frac{1}{2}\rho_t V_{T_t}^{\,2} S_W C_L} = \frac{W_s}{W_t} \tag{A2-15}$$

Hence,

$$\frac{W_t}{W_s}\,\frac{\overline{D}_s}{\overline{F}_t - \overline{D}_t} = \frac{\overline{D}_t}{\overline{F}_t - \overline{D}_t} = -1 + \frac{\overline{F}_t}{\overline{F}_t - \overline{D}_t} \tag{A2-16}$$

Substituting from (A2-14) and (A2-16) into (A2-12) we have

$$\frac{S_{a_t}}{S_{a_s}} = \left[\frac{W_t}{W_s}\,\frac{\overline{F}_s}{\overline{F}_t - \overline{D}_t} + 1 - \frac{\overline{F}_t}{\overline{F}_t - \overline{D}_t}\right]\frac{h_{v_t} + 50}{\frac{W_s}{W_t}\,\frac{\sigma_t}{\sigma_s}\,h_{v_t} + 50}$$

$$= \left[h_{v_t} + 50 + \frac{h_{v_t} + 50}{\bar{F}_t - \bar{D}_t} \left(\frac{W_t}{W_s} \quad \bar{F}_s - \bar{F}_t \right) \right] \Bigg/ \left(\frac{W_s}{W_t} \frac{\sigma_t}{\sigma_s} h_{v_t} + 50 \right)$$

$$= \left[h_{v_t} + 50 + S_{a_t} \left(\frac{\bar{F}_s}{W_s} - \frac{\bar{F}_t}{W_t} \right) \right] \Bigg/ \left(\frac{W_s}{W_t} \frac{\sigma_t}{\sigma_s} h_{v_t} + 50 \right)$$

Inverting, this becomes

$$\frac{S_{a_s}}{S_{a_t}} = \frac{\frac{W_s}{W_t} \frac{\sigma_t}{\sigma_s} h_{v_t} + 50}{h_{v_t} + 50 + S_{a_t} \frac{\bar{F}_s}{W_s} - S_{a_t} \frac{\bar{F}_t}{W_t}} \qquad \text{(A2-17)}$$

Derivation of Equations (4-3) and (4-3a): We have (equation (A2-1)

$$S_g \propto \frac{W^2}{\sigma} \frac{1}{\bar{F} - \bar{D}} = K \frac{W^2}{\sigma} \frac{1}{\bar{F} - \bar{D}} \quad \text{say} \qquad \text{(A2-18)}$$

Now by definition, as $\bar{D}$ depends on W only (equation (A2-5) and hence, S_g varies only with W, σ and $\bar{F}$

$$\frac{dS_g}{S_g} = \frac{W}{S_g} \frac{\partial S_g}{\partial W} \frac{dW}{W} + \frac{\sigma}{S_g} \frac{\partial S_g}{\partial \sigma} \frac{d\sigma}{\sigma} + \frac{\bar{F}}{S_g} \frac{\partial S_g}{\partial \bar{F}} \frac{d\bar{F}}{\bar{F}} \qquad \text{(A2-19)}$$

But

$$\frac{\partial S_g}{\partial W} = 2K \frac{W}{\sigma} \frac{1}{\bar{F} - \bar{D}} + K \frac{W^2}{\sigma} \frac{1}{(\bar{F} - \bar{D})^2} \frac{\partial \bar{D}}{\partial W}$$

$$W \frac{\partial S_g}{\partial W} = 2K \frac{W^2}{\sigma} \frac{1}{\bar{F} - \bar{D}} + K \frac{W^2}{\sigma} \frac{1}{\bar{F} - \bar{D}} \frac{\bar{D}}{\bar{F} - \bar{D}} \frac{\partial \bar{D}}{\partial W}$$

$$= S_g \left\{ 2 + \frac{\overline{D}}{\overline{F} - \overline{D}} \; \frac{W}{\overline{D}} \frac{\partial \overline{D}}{\partial W} \right\}$$

That is

$$\frac{W}{S_g} \; \frac{\partial S_g}{\partial W} = 2 + \frac{\overline{D}}{\overline{F} - \overline{D}} \; \frac{W}{\overline{D}} \; \frac{\partial \overline{D}}{\partial W} = 2 + \frac{\overline{D}}{\overline{F} - \overline{D}} \qquad \text{(A2-20)}$$

Since

$$\overline{D} \propto W = K' W \qquad \text{say}$$

$$\frac{\partial \overline{D}}{\partial W} = K' = \frac{\overline{D}}{W}$$

$$\frac{W}{\overline{D}} \frac{\partial \overline{D}}{\partial W} = 1 \qquad \text{(A2-21)}$$

Also,

$$\sigma \; \frac{\partial S_g}{\partial \sigma} = - \; \sigma K \; \frac{W^2}{\sigma^2} \; \frac{1}{\overline{F} - \overline{D}}$$

$$= - K \; \frac{W^2}{\sigma^2} \; \frac{1}{\overline{F} - \overline{D}} = - \; S_g$$

$$\frac{\sigma}{S_g} \; \frac{\partial S_g}{\partial \sigma} = - 1 \qquad \text{(A2-22)}$$

Further,

$$\frac{\partial S_g}{\partial \overline{F}} = - K \; \frac{W^2}{\sigma} \; \frac{1}{(\overline{F} - \overline{D})^2} = - \; \frac{S_g}{\overline{F} - \overline{D}}$$

$$\frac{\overline{F}}{S_g}\frac{\partial S_g}{\partial \overline{F}} = -\frac{\overline{F}}{\overline{F}-\overline{D}} \tag{A2-23}$$

Substituting from (A2-20), (A2-22) into (A2-19), we have

$$\frac{dS_g}{S_g} = \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{dW}{W} - \frac{d\sigma}{\sigma} - \frac{\overline{F}}{\overline{F}-\overline{D}}\frac{d\overline{F}}{\overline{F}} \tag{A2-24}$$

This equation, being in differentials, is exact. To apply it to our purpose we approximate by substituting ΔS_g, ΔW for dS_g, dW and so on, giving the approximate equation

$$\frac{\Delta S_g}{S_g} = \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{\Delta W}{W} - \frac{\Delta\sigma}{\sigma} - \frac{\overline{F}}{\overline{F}-\overline{D}}\frac{\Delta\overline{F}}{\overline{F}} \tag{A2-25}$$

Writing in observed base values, this becomes

$$\frac{\Delta S_g}{S_{g_t}} = \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{\Delta W}{W_t} - \frac{\Delta\sigma}{\sigma_t} - \frac{\overline{F}}{\overline{F}-\overline{D}}\frac{\Delta\overline{F}}{\overline{F}_t} \tag{A2-25a}$$

When, as is usual, the weight correction is small, it will be more convenient and will introduce negligible error to substitute W_s for W_t, writing

$$\frac{\Delta S_g}{S_{g_t}} = \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{\Delta W}{W_s} - \frac{\Delta\sigma}{\sigma_t} - \frac{\overline{F}}{\overline{F}-\overline{D}}\frac{\Delta\overline{F}}{\overline{F}_t} \tag{A2-25b}$$

An alternative form is obtained by integrating equation (A2-24), assuming the coefficients of dW, $d\sigma$ and $d\overline{F}$ constant, giving

$$\log_e S_g = \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\log_e W - \log_e\sigma - \frac{\overline{F}}{\overline{F}-\overline{D}}\log_e \overline{F}$$

That is,

$$S_g \propto W^{\left(2 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)} \sigma^{-1} \; \overline{F}^{\,-\left(\frac{\overline{F}}{\overline{F} - \overline{D}}\right)}$$

$$= W^{\left(2 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)} \sigma^{-1} \; \overline{F}^{\,-\left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)}$$

and hence

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{\left(2 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)} \frac{\sigma_t}{\sigma_s} \left(\frac{\overline{F}_t}{\overline{F}_s}\right)^{\left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)} \qquad \text{(A2-26)}$$

<u>Derivation of Equations (4-5) and (4-5a)</u>: We have (equation (2-11) of the main text and (A2-11) of this Appendix),

$$S_a = \frac{50\ W}{\overline{F} - \overline{D}} \quad \frac{h_v + 50}{50}$$

We have already seen, while deriving equation (2-12), that (equations (A2-14) and (A2-15))

$$h_v \propto \frac{W}{\sigma} = K'' \frac{W}{\sigma} \qquad \text{say}$$

$$\overline{D} \propto W = K''' \ W \qquad \text{say}$$

Hence,

$$\frac{\partial h_v}{\partial W} = \frac{K''}{\sigma} = \frac{h_v}{W}$$

$$\frac{W}{h_v} \frac{\partial h_v}{\partial W} = 1 \qquad \text{(A2-27)}$$

Similarly,

$$\frac{W}{\overline{D}} \frac{\partial \overline{D}}{\partial W} = 1 \qquad \text{(A2-28)}$$

$$\frac{\sigma}{h_v} \frac{\partial h_v}{\partial \sigma} = -1 \qquad \text{(A2-29)}$$

We may regard S_a as dependent on W, $\bar{F}$ and σ only, and write

$$\frac{dS_a}{S_a} = \frac{W}{S_a} \frac{\partial S_a}{\partial W} \frac{dW}{W} + \frac{\sigma}{S_a} \frac{\partial S_a}{\partial \sigma} \frac{d\sigma}{\sigma} + \frac{\bar{F}}{S_a} \frac{\partial S_a}{\partial \bar{F}} \frac{d\bar{F}}{\bar{F}} \qquad \text{(A2-30)}$$

Now,

$$\frac{\partial S_a}{\partial W} = \frac{50}{\bar{F} - \bar{D}} \frac{h_v + 50}{50} + \frac{50W}{(\bar{F} - \bar{D})^2} \frac{h_v + 50}{50} \frac{\partial \bar{D}}{\partial W}$$

$$+ \frac{50W}{\bar{F} - \bar{D}} \frac{1}{50} \frac{\partial h_v}{\partial W}$$

$$W \frac{\partial S_a}{\partial W} = \frac{50W}{\bar{F} - \bar{D}} \frac{h_v + 50}{50} \left\{ 1 + \frac{\bar{D}}{\bar{F} - \bar{D}} \frac{W}{\bar{D}} \frac{\partial \bar{D}}{\partial W} + \frac{h_v}{h_v + 50} \frac{W}{h_v} \frac{\partial h_v}{\partial W} \right\}$$

$$= S_a \left\{ 1 + \frac{\bar{D}}{\bar{F} - \bar{D}} \frac{W}{\bar{D}} \frac{\partial \bar{D}}{\partial W} + \frac{h_v}{h_v + 50} \frac{W}{h_v} \frac{\partial h_v}{\partial W} \right\}$$

Substituting from equations (A2-27) and (A2-29) and dividing both sides by S_a, we thus have

$$\frac{W}{S_a} \frac{\partial S_a}{\partial W} = 1 + \frac{\bar{D}}{\bar{F} - \bar{D}} + \frac{h_v}{h_v + 50} \qquad \text{(A2-31)}$$

Further,

$$\frac{\partial S_a}{\partial \sigma} = \frac{50W}{\overline{F} - \overline{D}} \frac{1}{50} \frac{\partial h_v}{\partial \sigma}$$

$$\sigma \frac{\partial S_a}{\partial \sigma} = \frac{50W}{\overline{F} - \overline{D}} \frac{h_v + 50}{50} \frac{h_v}{h_v + 50} \frac{\sigma}{h_v} \frac{\partial h_v}{\partial \sigma}$$

$$= S_a \frac{h_v}{h_v + 50} \frac{\sigma}{h_v} \frac{\partial h_v}{\partial \sigma}$$

Substituting from equation (A2-29) and dividing both sides by S_a we have

$$\frac{\sigma}{S_a} \frac{\partial S_a}{\partial \sigma} = - \frac{h_v}{h_v + 50} \tag{A2-32}$$

Also,

$$\frac{\partial S_a}{\partial \overline{F}} = - \frac{50W}{(\overline{F} - \overline{D})^2} \frac{h_v + 50}{50}$$

$$= - \frac{50W}{\overline{F} - \overline{D}} \frac{h_v + 50}{50} \frac{1}{\overline{F} - \overline{D}}$$

$$= - \frac{S_a}{\overline{F} - \overline{D}}$$

Hence,

$$\frac{\overline{F}}{S_a} \frac{\partial S_a}{\partial \overline{F}} = - \frac{\overline{F}}{\overline{F} - \overline{D}} = -1 - \frac{\overline{D}}{\overline{F} - \overline{D}} \tag{A2-33}$$

Substituting from equations (A2-31) thru (A2-33) into equation (A2-30) we thus have

$$\frac{dS_a}{S_a} = \left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}} + \frac{h_v}{h_v + 50}\right)\frac{dW}{W}$$

$$- \frac{h_v}{h_v + 50}\,\frac{d\sigma}{\sigma}$$

$$- \left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)\frac{d\overline{F}}{\overline{F}} \qquad \text{(A2-34)}$$

This equation, like equation (A2-24) for the ground roll, is exact. As in that case, we may now approximate and write

$$\frac{\Delta S_a}{S_{a_t}} = \left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}} + \frac{h_v}{h_v + 50}\right)\frac{\Delta W}{W_t}$$

$$- \frac{h_v}{h_v + 50}\,\frac{\Delta\sigma}{\sigma_t}$$

$$- \left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)\frac{\Delta \overline{F}}{\overline{F}_t} \qquad \text{(A2-35)}$$

For small weight corrections (say, less than 5%) it is again permissible to write W_s for W_t.

Alternatively, we may use an equation analogous to equation (A2-26). This is, by analogy

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{\left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}} + \frac{h_v}{h_v + 50}\right)} \left(\frac{\sigma_t}{\sigma_s}\right)^{\left(\frac{h_v}{h_v + 50}\right)} \left(\frac{\overline{F}_t}{\overline{F}_s}\right)^{\left(1 + \frac{\overline{D}}{\overline{F} - \overline{D}}\right)} \qquad \text{(A2-3}$$

APPENDIX III

The Transition Phase After a Take-Off at the Minimum Safe Speed

INTRODUCTION:

If the airplane takes off at the minimum safe speed, it has at that moment no excess of lift over weight with which to change the direction of motion. However, as the airplane accelerates the lift available without exceeding the maximum safe lift coefficient increases, and it becomes possible to pull the airplane up into a climb. This maneuver can be continued, using the maximum safe lift coefficient throughout, until the airplane either reaches 50 ft., or the angle at which it can climb steadily.

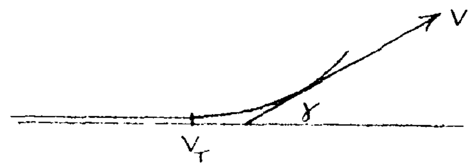
These maneuvers give an idea of the likely increase in airspeed between a minimum speed take-off, and the 50 ft. screen. Both are considered below.

NOTATION:

Symbol	Definition
h_v	$(V_{50}^2 - V_T^2)/2g$
s_a	distance from take-off to 50 ft.
t	time
t_c	time to attain steady climb from take-off
t_{50}	time to 50 ft. from take-off
V	true speed
V_T	take-off speed
V_{50}	speed at 50 ft.
α	(thrust-drag)/(weight of airplane)
γ	angle of climb
ξ	$2g\, s_a/V_T^2$
ω	$2g/V_T$

The Equations of Motion: We will assume that the thrust and drag remain constant throughout the transition from take-off to steady climb. This is a rather severe assumption for those cases for which the transition continues up to around 50 ft. altitude, but is necessary to permit a reasonably simple treatment.

Suppose that at a given instant, the airplane speed and angle of climb are V and γ. With the above assumption of constant thrust and drag, the excess thrust will be constant, equal to αW say.



Then

$$\frac{dV}{dt} = g(\alpha - \gamma) \tag{A3-1}$$

$$V\frac{d\gamma}{dt} = \text{normal acceleration}$$

$$= g\left\{\left(\frac{V}{V_T}\right)^2 - 1\right\} \tag{A3-2}$$

from consideration of the lift available.

Differentiating (A3-2) we have

$$\frac{dV}{dt}\,\frac{d\gamma}{dt} + V\frac{d^2\gamma}{dt^2} = 2g\,\frac{V}{V_T^2}\,\frac{dV}{dt}$$

$$\frac{d^2\gamma}{dt^2} = \frac{dV}{dt}\left\{\frac{2g}{V_T^2} - \frac{1}{V}\,\frac{d\gamma}{dt}\right\} \tag{A3-3}$$

Substituting from (A3-1) and (A3-2) for $\frac{dV}{d_t}$ and $\frac{d\gamma}{d_t}$ we then have

$$\frac{d^2\gamma}{dt^2} = g(\alpha - \gamma)\left\{\frac{2g}{V_T^2} - g\left(\frac{1}{V_T^2} - \frac{1}{V^2}\right)\right\}$$

$$= 2\left(\frac{g}{V_T}\right)^2(\alpha - \gamma)\left\{1 - \frac{1}{2}\left(1 - \frac{V_T^2}{V^2}\right)\right\} \qquad \text{(A3-4)}$$

$$= 2\left(\frac{g}{V_T}\right)^2(\alpha - \gamma) \qquad \text{approximately} \qquad \text{(A3-5)}$$

This equation indicates a sinusoidal motion about $\frac{d^2\gamma}{dt^2} = 2\left(\frac{g}{V_T}\right)^2\alpha$. We will follow this until we reach either the angle at which steady climb is possible or 50 ft., whichever occurs first.

As a solution to (A3-5) we can write

$$\gamma = \alpha\left\{1 - \cos\omega t\right\} \qquad \text{(A3-6)}$$

since γ is initially zero when t is zero

$$\frac{d^2\gamma}{dt^2} = \alpha\omega^2\cos\omega t = \omega^2(\alpha - \gamma)$$

Hence

$$\omega = \sqrt{2}\,\frac{g}{V_T} \qquad \text{(A3-7)}$$

<u>Steady Climb Attained Before 50 ft.</u>: If, now, we further assume that the possible steady angle of climb is equal to α i.e. that the excess thrust is still unchanged, we see that the airplane attains the steady climb angle when

$$\cos\omega t_c = 0$$
$$\omega t_c = \frac{\pi}{2} \qquad \text{(A3-8)}$$

$$t_c = \frac{\pi}{2\omega} = \frac{V_T}{g}\,\frac{\pi}{2\sqrt{2}} \tag{A3-9}$$

During this time, we have

$$\text{distance covered} \simeq t\,V_T = \frac{V_T^2}{g}\,\frac{\pi}{2\sqrt{2}}$$

$$\text{height gained} = V_T \int_0^{t_c} \gamma\, dt$$

$$= V_T \alpha \int_0^{t_c} (1 - \cos \omega t)\, dt$$

$$= V_T \alpha \left[t - \frac{1}{\omega} \sin \omega t \right]_0^{t_c}$$

$$= V_T \alpha t_c \left\{ 1 - \frac{2}{\pi} \right\} \tag{A3-10}$$

Hence we have total distance to 50 ft., S_a, is equal to

$$\frac{1}{\alpha} \left\{ 50 - V_T \alpha\, t_c \left(1 - \frac{2}{\pi} \right) \right\} + \frac{V_T^2}{g}\,\frac{\pi}{2\sqrt{2}}$$

$$= \frac{50}{\alpha} - \frac{V_T^2}{g}\,\frac{\pi}{2\sqrt{2}} \left(1 - \frac{2}{\pi} \right) + \frac{V_T^2}{g}\,\frac{\pi}{2\sqrt{2}}$$

$$= \frac{50}{\alpha} + \frac{V_T^2}{g\sqrt{2}} \tag{A3-11}$$

The speed on the steady climb is then

$$V_T + \int_{V_T}^{V_{50}} dV = V_T + \int_0^{t_c} \frac{dV}{dt}\, dt$$

$$= V_T + \int_0^{t_c} \alpha g \cos \omega t \, dt$$

$$= V_T + \alpha g \left[\frac{1}{\omega} \sin \omega t \right]_0^{t_c}$$

$$= V_T + \alpha g \times \frac{V_T}{\sqrt{2}\, g}$$

$$= V_T \left(1 + \frac{\alpha}{\sqrt{2}} \right) \tag{A3-12}$$

For the kinetic energy increase between take-off and 50 ft., we have

$$h_v = \frac{V_{50}^2 - V_T^2}{2g} = \frac{V_T^2}{2g} \frac{\alpha}{\sqrt{2}} \left(2 + \frac{\alpha}{\sqrt{2}} \right)$$

$$= \alpha \frac{V_T^2}{\sqrt{2}\, g} \qquad \text{approximately} \tag{A3-13}$$

But, we also have, considering the total energy increase,

$$h_v + 50 = \frac{V_{50}^2 - V_T^2}{2g} + 50$$

$$= \alpha S_a \tag{A3-14}$$

Dividing equation (A3-13) by equation (A3-14), we have

$$\frac{h_v}{h_v + 50} = \frac{1}{\sqrt{2}} \frac{V_T^2}{g\, S_a} \qquad \text{(A3-15)}$$

In the above analysis, any increase in induced drag due to curvature of the flight path, is ignored. It is shown in Appendix II that this increase will increase the air distance by less than about

$$1.2 \frac{V_T^2}{g} \frac{D_{min}}{W}$$

$$\doteq 0.1 \frac{V_T^2}{g} \qquad \text{say}$$

Thus, this effect will change the transition distance by less than 15%. As the present analysis is approximate only, being designed to indicate the order of and shape of the effects investigated, this is considered good enough.

<u>Transition Incomplete at 50 ft.</u>: From equation (A3-10) the height gained during transition to steady climb will be exactly 50 ft. when

$$50/\alpha = V_T t_c \left\{ 1 - \frac{2}{\pi} \right\}$$

Substituting from equation (A3-11),

$$S_a - \frac{V_T^2}{g\sqrt{2}} = V_T t_c \left\{ 1 - \frac{2}{\pi} \right\}$$

$$= \frac{V_T^2}{g} \frac{\pi}{2\sqrt{2}} \left\{ 1 - \frac{2}{\pi} \right\} \qquad \text{from equation (A3-9)}$$

$$= \frac{V_T^2}{g} \frac{\pi}{2\sqrt{2}} - \frac{V_T^2}{g\sqrt{2}}$$

i.e. $$S_a = \frac{V_T^2}{g} \frac{\pi}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} \frac{V_T^2}{g\, S_a} = \frac{2}{\pi} \qquad \text{(A3-16)}$$

Under these conditions, we have (equation A3-15)

$$\frac{h_v}{h_v + 50} = \frac{1}{\sqrt{2}} \frac{V_T^2}{g S_a} = \frac{2}{\pi} = 0.64 \qquad \text{(A3-17)}$$

For higher values of $V_T^2/(\sqrt{2}g\ S_a)$ transition is incomplete at 50 ft. At the 50 ft. point, we then have

50 = height gained

$$= \int_0^{t_{50}} V_T \gamma \, dt \qquad \text{approximately}$$

$$= V_T \alpha \int_0^{t_{50}} (1 - \cos\omega t)\, dt$$

$$= V_T \alpha \left[t - \frac{1}{\omega} \sin\omega t \right]_0^{t_{50}}$$

$$= V_T \alpha t_{50} \left(1 - \frac{\sin\omega t_{50}}{\omega t_{50}} \right) \qquad \text{(A3-18)}$$

But the air distance S_a is approximately given by

$$S_a = V_T t_{50} \tag{A3-19}$$

Also, from equation (A3-7)

$$\omega = \sqrt{2}\,\frac{g}{V_T}$$

Hence, we have

$$50 = \alpha S_a \left\{ 1 - \frac{\sin\left(\sqrt{2}\, g \frac{S_a}{V_T^2}\right)}{\sqrt{2}\, g \frac{S_a}{V_T^2}} \right\} \tag{A3-20}$$

$$= \alpha S_a \left\{ 1 - \frac{\sin \xi}{\xi} \right\} \tag{A3-21}$$

Considering now, the speed at 50 ft., we have

$$V_{50} - V_T = \int_0^{t_{50}} \frac{dV}{dt}\, dt$$

$$= \int_0^{t_{50}} \alpha g \cos \omega t \, dt$$

$$= \alpha g \left[\frac{1}{\omega} \sin \omega t \right]_0^{t_c}$$

$$= \alpha g \frac{V_T}{\sqrt{2}\, g} \sin \omega t_c$$

$$= \alpha g \frac{V_T}{\sqrt{2}\, g} \sin \left(\sqrt{2}\, \frac{g S_a}{V_T^2} \right)$$

$$V_{50} = V_T \left(1 - \frac{\alpha}{\sqrt{2}} \frac{\sin \xi}{\xi} \right) \tag{A3-22}$$

$$h_v = \frac{V_{50}^2 - V_T^2}{2g}$$

$$= \frac{V_T^2}{g}\left(\frac{V_{50}}{V_T} - 1\right) \qquad \text{approximately}$$

$$= \frac{V_T^2}{g}\,\frac{\alpha}{2}\,\sin\xi \qquad \text{from equation (A3-22)}$$

But $h_v + 50$ = total energy increase/W

$$= \alpha\, S_a$$

Hence,

$$\frac{h_v}{h_v + 50} = \frac{V_T^2}{\sqrt{2}\, g\, S_a}\,\sin\xi$$

$$= \frac{\sin\xi}{\xi} \quad \text{where } \xi = \sqrt{2}\, g\,\frac{S_a}{V_T^2} \qquad \text{(A3-23)}$$

Equation (A3-15) can now be written

$$\frac{h_v}{h_v + 50} = \frac{1}{\xi} \qquad \text{(A3-24)}$$

Equation (A3-24) applies for values of ξ down to $\frac{\pi}{2}$, after which equation (A3-23) should be used.

Using these relations $\frac{h_v}{h_v + 50}$ is plotted against $V_T^2/g\,S_a$ in Figure 1A.

APPENDIX IV

Correction of Propeller Thrust to Standard Conditions

NOTATION:

Symbol	Definition
C_{L_T}	lift coefficient at take-off
C_P	$P/\rho n^3 d^5$
C_Q	$Q/\rho n^2 d^5$
C_T	$F/\rho n^2 d^4$
$\bar{C}_T$	value of C_T when $V = \bar{V}$
d	propeller diameter
F	thrust
$\bar{F}$	thrust at mean speed $\bar{V}$ for phase
J	V/nd
$\bar{J}$	$\bar{V}/nd$
n	propeller rotational speed
N	engine rotational speed
P	power input to propeller
P_a	ambient air pressure
Q	torque
V	true speed
V_T	true speed at take-off
V_{50}	true speed at 50 ft.
$\bar{V}$	mean speed during phase
W	airplane gross weight

NOTATION (CONTINUED):

Symbol	Definition
η	propeller efficiency
η_t	value of η given by simple momentum theory
ρ	air density
σ	air density/standard Sea Level density

Subscripts o, s, t denote to static, standard and test conditions respectively.

Constant Speed Propellers: If there are no compressibility effects on efficiency we may write

$$\eta = f(C_P, J)$$

$$\text{where } C_P = P/\rho n^3 d^5$$

$$J = V/nd$$

$$d = \text{propeller diameter}$$

$$n = \text{propeller rotational speed}$$

and a consistent system of units is used. That is, η is a function of C_P and J only. Hence, by definition,

$$d\eta = \frac{\partial \eta}{\partial C_P} dC_P + \frac{\partial \eta}{\partial J} dJ \qquad \text{(A4-1)}$$

$$\frac{d\eta}{\eta} = \frac{C_P}{\eta}\frac{\partial \eta}{\partial C_P}\frac{dC_P}{C_P} + \frac{J}{\eta}\frac{\partial \eta}{\partial J}\frac{dJ}{J} \qquad \text{(A4-2)}$$

We also have, by definition

$$F = \frac{\eta P}{V}$$

$$dF = \frac{P}{V}\,d\eta + \frac{\eta}{V}\,dP - \frac{\eta P}{V^2}\,dV$$

$$= \frac{\eta P}{V} \times \frac{d\eta}{\eta} + \frac{\eta P}{V} \times \frac{dP}{P} - \frac{\eta P}{V}\,\frac{dV}{V}$$

$$= F\left(\frac{d\eta}{\eta} - \frac{dP}{P} - \frac{dV}{V}\right)$$

Hence,

$$\frac{dF}{F} = \frac{d\eta}{\eta} + \frac{dP}{P} - \frac{dV}{V} \qquad (A4\text{-}3)$$

Further,

$$C_P = \frac{P}{\rho n^3 d^5}$$

$$dC_P = \frac{dP}{\rho n^3 d^5} - \frac{P\,d\rho}{\rho^2 n^3 d^5} - 3\,\frac{P\,dn}{\rho n^4 d^5}$$

$$= \frac{P}{\rho n^3 d^5}\left\{\frac{dP}{P} - \frac{d\rho}{\rho} - 3\,\frac{dn}{n}\right\}$$

i.e.

$$\frac{dC_P}{C_P} = \frac{dP}{P} - \frac{d\sigma}{\sigma} - 3\,\frac{dN}{N} \qquad (A4\text{-}4)$$

where σ = relative density, proportional to ρ

N = engine speed, proportional to propeller speed

Similarily,

$J = V/nd$

$$dJ = \frac{dV}{nd} - \frac{Vdn}{n^2 d}$$

$$= \frac{V}{nd}\left(\frac{dV}{V} - \frac{dn}{n}\right)$$

$$\frac{dJ}{J} = \frac{dV}{V} - \frac{dn}{n} = \frac{dV}{V} - \frac{dN}{N} \qquad \text{(A4-5)}$$

Substituting from (A4-4) and (A4-5) into (A4-2) we have, after regrouping,

$$\frac{d\eta}{\eta} = \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\frac{dP}{P} - \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\frac{d\sigma}{\sigma}$$

$$- \left(3\frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P} + \frac{J}{\eta}\frac{\partial\eta}{\partial J}\right)\frac{dN}{N} + \frac{J}{\eta}\frac{\partial\eta}{\partial J}\frac{dV}{V} \qquad \text{(A4-6)}$$

Substituting from (A4-6) into (A4-3), we have

$$\frac{dF}{F} = \left(1 + \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\right)\frac{dP}{P} - \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\frac{d\sigma}{\sigma}$$

$$- \left(3\frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P} + \frac{J}{\eta}\frac{\partial\eta}{\partial J}\right)\frac{dN}{N}$$

$$+ \left(\frac{J}{\eta}\frac{\partial\eta}{\partial J} - 1\right)\frac{dV}{V} \qquad \text{(A4-7)}$$

Considering now the mean thrust $\bar{F}$ in each phase at speed $\bar{V}$, we have for the ground phase

$$\bar{V} = 0.75\, V_r = 0.75\sqrt{\frac{W}{\frac{1}{2}\rho S_W C_{L_T}}}$$

$$\propto \sqrt{\frac{W}{\sigma}} = K_G\sqrt{\frac{W}{\sigma}} \quad \text{say} \qquad \text{(A4-8)}$$

and for the air phase

$$\overline{V} = V_{50} = \sqrt{\frac{W}{\frac{1}{2}\rho S_W C_{L_{50}}}} \quad \propto \quad \sqrt{\frac{W}{\sigma}}$$

$$= K_{50}\sqrt{\frac{W}{\sigma}} \quad \text{say} \tag{A4-9}$$

In the first case

$$d\overline{V} = \frac{1}{2}\,\frac{K_G}{\sqrt{\sigma}}\,\frac{dW}{\sqrt{W}} - \frac{1}{2}\,K_G\sqrt{W}\,\frac{d\sigma}{\sigma^{3/2}}$$

$$= K_G\sqrt{\frac{W}{\sigma}}\left\{\frac{1}{2}\,\frac{dW}{W} - \frac{1}{2}\,\frac{d\sigma}{\sigma}\right\}$$

$$\frac{d\overline{V}}{\overline{V}} = \frac{1}{2}\,\frac{dW}{W} - \frac{1}{2}\,\frac{d\sigma}{\sigma} \tag{A4-10}$$

By inspection of equations (A4-8), from which the above equation was deduced, and equation (A4-9), it will be seen that the same relation will hold for the air phase. Hence, substituting in (A4-7) we have for the mean thrust $\overline{F}$ at speed $\overline{V}$.

$$\frac{d\overline{F}}{\overline{F}} = \left(1 + \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\right)\frac{dP}{P} - \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\frac{d\sigma}{\sigma}$$

$$- \left(3\,\frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P} + \frac{J}{\eta}\frac{\partial\eta}{\partial J}\right)\frac{dN}{N} + \left(\frac{J}{\eta}\frac{\partial\eta}{\partial J} - 1\right)\left(\frac{1}{2}\frac{dW}{W} - \frac{1}{2}\right.$$

$$= \left(1 + \frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P}\right)\frac{dP}{P}$$

$$- \left(\frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P} + \frac{1}{2}\,\frac{J}{\eta}\frac{\partial\eta}{\partial J} - \frac{1}{2}\right)\frac{d\sigma}{\sigma}$$

$$- \left(3\,\frac{C_P}{\eta}\frac{\partial\eta}{\partial C_P} + \frac{J}{\eta}\frac{\partial\eta}{\partial J}\right)\frac{dN}{N}$$

$$- \frac{1}{2}\left(\frac{J}{\eta}\frac{\partial\eta}{\partial J} - 1\right)\frac{dW}{W} \tag{A4-11}$$

$$= A_P\frac{dP}{P} + A_\sigma\frac{d\sigma}{\sigma} + A_N\frac{dN}{N} + A_W\frac{dW}{W} \quad \text{say}$$

Approximating, we may therefore, write,

$$\frac{\Delta \bar{F}}{F_t} = A_P \frac{\Delta P}{P_t} + A_\sigma \frac{\Delta \sigma}{\sigma_t} + A_N \frac{\Delta N}{N_t} + A_V \frac{\Delta V}{V_t} \qquad \text{(A4-12)}$$

Values of Derivatives of Propeller Efficiency: Much of the energy loss in the propeller at take-off consists of kinetic energy transmitted to the air flowing through the propeller disc. It is, therefore, possible that simple momentum theory may give some idea of how the propeller will behave.

If η_f is the propeller efficiency given by accounting only for this lost kinetic energy (the "Froude Efficiency") it is shown in Chapter XV of Reference 1, that

$$\frac{1-\eta_f}{\eta_f^3} = \frac{2}{\pi} \frac{C_P}{J^3} \qquad \text{(A4-13)}$$

Differentiating this equation

$$-3\frac{d\eta_f}{\eta_f^4} + 2\frac{d\eta_f}{\eta_f^3} = \frac{2}{\pi}\frac{dC_P}{J^3} - 3\frac{2}{\pi}\frac{C_P}{J^4}dJ$$

$$\text{i.e.} \quad \frac{d\eta_f}{\eta_f}\left(-\frac{3}{\eta_f^3} + \frac{2}{\eta_f^2}\right) = \frac{2}{\pi}\frac{C_P}{J^3}\left(\frac{dC_P}{C_P} - 3\frac{dJ}{J}\right)$$

Dividing by equation (A4-13) we have,

$$\frac{d\eta_f}{\eta_f}\left(\frac{-3+2\eta_f}{1-\eta_f}\right) = \frac{dC_P}{C_P} - 3\frac{dJ}{J}$$

$$\text{or} \quad \frac{d\eta_f}{\eta_f} = \frac{1-\eta_f}{2\eta_f-3}\left(\frac{dC_P}{C_P} - 3\frac{dJ}{J}\right)$$

$$= -\left(\frac{1}{3}\frac{dC_P}{C_P} - \frac{dJ}{J}\right)\left(1 - \frac{\eta_f}{3-2\eta_f}\right) \qquad \text{(A4-14)}$$

But by definition

$$d\eta_s = \frac{\partial \eta_s}{\partial C_P}\, dC_P + \frac{\partial \eta_s}{\partial J}\, dJ$$

$$\frac{d\eta_s}{\eta_s} = \frac{C_P}{\eta_s}\frac{\partial \eta_s}{\partial C_P}\frac{dC_P}{C_P} + \frac{J}{\eta_s}\frac{\partial \eta_s}{\partial J}\frac{dJ}{J} \qquad \text{(A4-15)}$$

The coefficients of $\frac{dC_P}{C_P}$ and $\frac{dJ}{J}$ in equations (A4-14) and (A4-15) must be identical, so we have

$$\frac{C_P}{\eta_s}\frac{\partial \eta_s}{\partial C_P} = -3\left(1 - \frac{\eta_s}{3 - 2\eta_s}\right) \qquad \text{(A4-16)}$$

$$\frac{J}{\eta_s}\frac{\partial \eta_s}{\partial J} = 1 - \frac{\eta_s}{3 - 2\eta_s} \qquad \text{(A4-17)}$$

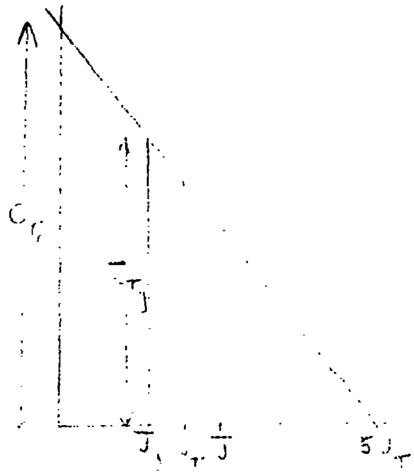
In Figure 3, the curve for A_J is that given by assuming that equation (A2-17) applies to the actual efficiency η. Computed values of $\frac{J}{\eta}\frac{\partial \eta}{\partial J}$ for a wide range of propellers agrees with this curve to within 0.2, which is near enough for our purposes. Also, in the curves for A_P and A_J, the base lines (not labeled for particular J) are derived from equations (A2-16) and (A2-17). The branches labeled for particular J are mean curves derived from computations for a wide range of propellers, based on NACA data.

Fixed Pitch Propellers: It is stated in Section 7.3 of the main text, that it is assumed that for fixed pitch propellers in the take-off range

(a) the torque coefficient C_Q is constant

(b) the thrust coefficient C_T varies linearly with advance diameter ratio J, becoming zero at five times the take-off speed.

First let us consider the variation of $\overline{C}_T$, the thrust coefficient at the mean speed, with $\overline{J}$. From the diagram we see that assumption (b) gives



$$\frac{d\overline{C}_T}{d\overline{J}} = -\frac{C_{T_0}}{5J_T}$$

$$\frac{\overline{J}}{\overline{C}_T}\frac{\partial\overline{C}_T}{\partial\overline{J}} = -\frac{1}{5}\frac{C_{T_0}}{\overline{C}_T}\frac{\overline{J}}{J_T}$$

$$= -\frac{\overline{J}}{5J_T - \overline{J}}$$

For the ground phase $J = 0.75\ J_T$ and hence

$$\frac{\overline{J}}{\overline{C}_T}\frac{d\overline{C}_T}{d\overline{J}} = -\frac{0.75}{4.25} = -0.18 \qquad \text{(A4-18)}$$

For the air phase we may take as an average value $\overline{J} = 1.1\ J_T$ giving

$$\frac{\overline{J}}{\overline{C}_T}\frac{d\overline{C}_T}{d\overline{J}} = -\frac{1.1}{3.9} = -0.28 \qquad \text{(A4-19)}$$

In general, there may be a change in engine speed between test and standard conditions. By definition

$$\overline{F} = \rho n^2 d^4 \overline{C}_T \qquad \text{(A4-20)}$$

$$d\overline{F} = n^2 d^4 \overline{C}_T\, d\rho + 2\rho n d^4 \overline{C}_T\, dn + \rho n^2 d^4\, d\overline{C}_T$$

$$= \rho n^2 d^4 \overline{C}_T \left(\frac{d\rho}{\rho} + 2\frac{dn}{n} + \frac{d\overline{C}_T}{\overline{C}_T} \right)$$

$$= \overline{F}\left(\frac{d\sigma}{\sigma} + 2\frac{dN}{N} - \frac{d\overline{C}_T}{\overline{C}_T} \right) \qquad \text{(A4-21)}$$

But from equations (A4-18) and (A4-19)

$$\frac{d\bar{C}_T}{\bar{C}_T} = -0.18\,\frac{d\bar{J}}{\bar{J}}$$ (A4-22) for ground phase

$$\frac{d\bar{C}_T}{\bar{C}_T} = -0.28\,\frac{d\bar{J}}{\bar{J}}$$ (A4-23) for air phase

From equation (A4-5) and (A4-10)

$$\frac{d\bar{J}}{\bar{J}} = \frac{d\bar{V}}{\bar{V}} - \frac{dN}{N}$$

$$= \frac{1}{2}\,\frac{dW}{W} - \frac{1}{2}\,\frac{d\sigma}{\sigma} - \frac{dN}{N} \qquad \text{(A4-24)}$$

Hence from (A2-22) and (A2-23)

$$\frac{d\bar{C}_T}{\bar{C}_T} = -0.09\,\frac{dW}{W} + 0.09\,\frac{d\sigma}{\sigma} - 0.18\,\frac{dN}{N}$$ for the ground phase (A4-25)

$$\frac{d\bar{C}_T}{\bar{C}_T} = -0.14\,\frac{dW}{W} + 0.14\,\frac{d\sigma}{\sigma} - 0.28\,\frac{dN}{N}$$ for the air phase (A4-26)

Substituting in equation (A2-21) we have for the ground phase

$$\frac{dF}{F} = \frac{d\sigma}{\sigma}\left(1 + 0.09\right) + \frac{dN}{N}\left(2 - 0.18\right) - 0.09\,\frac{dW}{W}$$

$$= 1.09\,\frac{d\sigma}{\sigma} + 1.82\,\frac{dN}{N} - 0.09\,\frac{dW}{W} \qquad \text{(A4-27)}$$

and for the air phase

$$\frac{d\bar{F}}{\bar{F}} = 1.14\frac{d\sigma}{\sigma} + 1.72\frac{dN}{N} - 0.14\frac{dW}{W} \tag{A4-28}$$

<u>Correction at Constant Engine Speed</u>: If the engine is at maximum permissible speed under both test and standard conditions $\frac{dN}{N}$ is zero. Rounding off equations (A4-27) and (A4-28) we then find that the following equation may be applied to both ground and air phases

$$\frac{d\bar{F}}{\bar{F}} = 1.1\frac{d\sigma}{\sigma} - 0.1\frac{dW}{W} \tag{A4-29}$$

Approximately, we may therefore write

$$\frac{\Delta\bar{F}}{\bar{F}_t} = 1.1\frac{\Delta\sigma}{\sigma_t} - 0.1\frac{\Delta W}{W_t} \tag{A4-30}$$

<u>Correction at Full Throttle</u>: If the engine is at full throttle its rotational speed will change with air temperature. We have, from assumption (a) of paragraph 5.3 of the main text

$$C_Q = \frac{Q}{\rho n^2 d^5} = \text{constant}$$

where Q = torque

Hence

$$Q \propto \rho n^2$$

$$\propto \frac{P_a}{T_a} N^2 \tag{A4-31}$$

where P_a = ambient air pressure

T_a = ambient air temperature

We will further assume that at full throttle,

$$Q \propto \frac{T_a}{\sqrt{T_a}} \qquad \text{(A4-32)}$$

Combining (A4-31) and (A4-32)

$$N^2 \frac{P_a}{T_a} \propto \frac{P_a}{\sqrt{T_a}}$$

$$N \propto T_a^{\frac{1}{4}} = K\, T_a^{\frac{1}{4}} \quad \text{say}$$

$$dN = \frac{1}{4} K\, T_a^{-\frac{3}{4}}$$

$$\frac{dN}{N} = \frac{1}{4} \frac{dT_a}{T_a} \qquad \text{(A4-33)}$$

Substituting in equations (A4-27) and (A4-28) and rounding off, we may write, for both ground and air phases

$$\frac{d\bar{F}}{\bar{F}} = 1.1 \frac{d\sigma}{\sigma} + 0.4 \frac{dT_a}{T_a} - 0.1 \frac{dW}{W}$$

or, in the approximate form

$$\frac{\Delta \bar{F}}{\bar{F}_t} = 1.1 \frac{\Delta\sigma}{\sigma_t} + 0.4 \frac{\Delta T_a}{T_{a_t}} - 0.1 \frac{\Delta W}{W_t} \qquad \text{(A4-34)}$$

APPENDIX V

Correction Formulae When ATO is Used Part-Time

NOTATION:

Symbol	Definition
$\overline{D}$	total resistance at speed $\overline{V}$
$\overline{F}$	effective total mean thrust
F_R	thrust of ATO units
$\overline{F}_R$	effective mean thrust of ATO units during phase
$\overline{F}_b$	mean basic thrust (without ATO)
$\mathfrak{R}$	$\overline{F}/\overline{F}_R$
S	distance covered during phase
S_g	ground roll
S_R	distance covered during phase with ATO operating
t_{R_a}	duration of ATO within air phase
t_{R_g}	duration of ATO in ground phase
V_T	take-off speed
$\overline{V}_R$	mean speed during part of ground roll with ATO operating
W	airplane gross weight
σ	air density/standard sea level air density

<u>Evaluation of Effective Mean Rocket Thrust</u>: To use the general equations connecting take-off ground roll and air distance with mean thrust it is desirable to substitute for the thrust of any ATO operated over part of the phase, a mean effective ATO thrust assumed to act throughout the relevant phase. To do so, we need a relation by which to approximate such a mean effective thrust from the test data.

An approach which shows immediate promise is to use a mean effective thrust which does the same work on the airplane. That is, to write

$$\bar{F}_R = \frac{S_R}{S} F_R \qquad \text{(A5-1)}$$

where F_R = thrust of ATO devices

S = length of phase of take-off

S_R = distance over which ATO operates during phase

$\bar{F}_R$ = mean effective thrust of ATO

We will show by representative examples, that this approximation is satisfactory for the ground roll case, in which a wide range of speed and possibly of acceleration is covered. It will then be presumed that the approximation is satisfactory for the air phase, over which the air speed does not vary much.

The following examples have been considered:

Basic excess thrust decreasing linearily with increase of air speed to

(i) 80% of its initial value

(ii) 20% of its initial value

with ATO fired at

(i) 20% of take-off speed

(ii) 80% of take-off speed

The assumed ATO thrust was equal to one-half of initial static thrust. The mean excess thrust was computed using equation (A5-1) and the ratio of the actual ground roll to that given by the relation

$$S_A = \frac{V_T^2}{2gW} \times \text{(mean excess thrust)}$$

has been computed, with results as follows:

Basic Thrust at T.O. / Basic Static Thrust	ATO Firing Speed / TO Speed	Approximate Ground Roll / Exact Ground Roll
0.8	0.2	1.003
	0.8	1.006
0.2	0.2	1.004
	0.8	1.06

It will be seen that the error is only appreciable in the extreme case in which the basic excess thrust falls off very sharply and the ATO is fired rather late in the run. So severe a fall in basic excess thrust would only occur with an overloaded propeller airplane. Even then, the error is not very large for purposes of data standardization and may be accepted.

<u>Evaluation of Change in Ground Roll</u>: From equation (A5-1), we have

$$\overline{F}_R = \frac{S_R}{S} F_R$$

$$= \frac{F_R}{S} \overline{V}_R \, t_{R_g} \qquad \text{(A5-2)}$$

where $\overline{V}_R$ = mean true speed during part of phase during which ATO is operating

t_{R_g} = duration of ATO during phase

Assuming F_R constant, we have for the ground phase

$$d\overline{F}_R = -\frac{F_R}{S_g} \overline{V}_R \, t_{R_g} \, dS_g + \frac{F_R}{S_g} t_{R_g} \, d\overline{V}_R + \frac{F_R}{S_g} \overline{V}_R \, dt_{R_g}$$

$$\frac{d\overline{F}_R}{\overline{F}_R} = -\frac{dS_g}{S_g} + \frac{d\overline{V}_R}{\overline{V}_R} + \frac{dt_{R_g}}{t_{R_g}} \qquad \text{(A5-3)}$$

We will now approximate and assume that $\overline{V}_R$ is proportional to $\sqrt{W/\sigma}$. This will be true to the first order.

Then

$$\frac{d\overline{V}_R}{\overline{V}_R} = \frac{1}{2}\frac{dW}{W} - \frac{1}{2}\frac{d\sigma}{\sigma} \tag{A5-4}$$

If $\overline{F}$ is the total mean thrust and $\overline{F}_b$ the mean thrust of the basic power plants, we already have (equation (A2-24) of Appendix II).

$$\frac{ds_g}{s_g} = \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{dW}{W} - \frac{d\sigma}{\sigma} - \left(1 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{d\overline{F}}{\overline{F}} \tag{A5-5}$$

$$= \left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{dW}{W} - \frac{d\sigma}{\sigma} - \left(1 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\left(\frac{d\overline{F}_b}{\overline{F}} + \frac{\overline{F}_R}{\overline{F}}\frac{d\overline{F}_R}{\overline{F}_R}\right) \tag{A5-6}$$

Substituting from (A5-4) and (A5-6) into (A5-3) we thus have

$$\frac{d\overline{F}_R}{\overline{F}_R} = -\left(2 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{dW}{W} + \frac{d\sigma}{\sigma} + \left(1 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\left(\frac{d\overline{F}_b}{\overline{F}} + \frac{\overline{F}_R}{\overline{F}}\frac{d\overline{F}_R}{\overline{F}_R}\right)$$
$$+ 0.5\frac{dW}{W} - 0.5\frac{d\sigma}{\sigma} + \frac{dt_{R_g}}{t_{R_g}}$$

$$\frac{d\overline{F}_R}{\overline{F}_R}\left\{1 - \frac{\overline{F}_R}{\overline{F}}\left(1 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\right\} = -\left(1.5 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{dW}{W} + 0.5\frac{d\sigma}{\sigma} + \frac{dt_{R_g}}{t_{R_g}}$$
$$+ \left(1 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{d\overline{F}_b}{\overline{F}}$$

i.e. $\frac{d\overline{F}_R}{\overline{F}}\left(\frac{\overline{F}}{\overline{F}_R} - 1 - \frac{\overline{D}}{\overline{F}-\overline{D}}\right)$

$$= -\left(1.5 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{dW}{W} + 0.5\frac{d\sigma}{\sigma} + \frac{dt_{R_g}}{t_{R_g}} + \left(1 + \frac{\overline{D}}{\overline{F}-\overline{D}}\right)\frac{d\overline{F}_b}{\overline{F}}$$

Hence

$$\frac{d\bar{F}}{\bar{F}} = \frac{d\bar{F}_R}{\bar{F}} + \frac{d\bar{F}_\ell}{\bar{F}}$$

$$= -\frac{1.5 + \dfrac{\bar{D}}{\bar{F}-\bar{D}}}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}}\,\frac{dW}{W} - \frac{0.5}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}}\,\frac{d\sigma}{\sigma}$$

$$- \frac{1}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}}\,\frac{dt_{R_3}}{t_{R_3}}$$

$$+ \left\{ 1 + \frac{1 + \dfrac{\bar{D}}{\bar{F}-\bar{D}}}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}} \right\} \frac{d\bar{F}_\ell}{\bar{F}} \qquad \text{(A5-7)}$$

Substituting into equation (A5-5) we have,

$$\frac{dS_3}{S_3} = \left\{ 2 + \frac{\bar{D}}{\bar{F}-\bar{D}} + \left(1 + \frac{\bar{D}}{\bar{F}-\bar{D}}\right) \frac{1.5 + \dfrac{\bar{D}}{\bar{F}-\bar{D}}}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}} \right\} \frac{dW}{W}$$

$$- \left\{ 1 + \left(1 + \frac{\bar{D}}{\bar{F}-\bar{D}}\right) \frac{0.5}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}} \right\} \frac{d\sigma}{\sigma}$$

$$- \left(1 + \frac{\bar{D}}{\bar{F}-\bar{D}}\right) \frac{1}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}}\,\frac{dt_{R_3}}{t_{R_3}}$$

$$- \left(1 + \frac{\bar{D}}{\bar{F}-\bar{D}}\right)\left(1 + \frac{1 + \dfrac{\bar{D}}{\bar{F}-\bar{D}}}{\dfrac{\bar{F}}{\bar{F}_R} - 1 - \dfrac{\bar{D}}{\bar{F}-\bar{D}}}\right) \frac{d\bar{F}_\ell}{\bar{F}} \qquad \text{(A5-8)}$$

Writing $\dfrac{\overline{D}}{\overline{F} - \overline{D}} = 0.3$, as is proposed in Section 5 of the main text, and writing ΔS_g for dS_g, etc., and $\mathcal{R}$ for $\dfrac{\overline{F}}{\overline{F}_R}$ this becomes

$$\frac{\Delta S_g}{S_g} = \left\{ 2.3 + \frac{2.3}{\mathcal{R} - 1.3} \right\} \frac{\Delta W}{W_t} - \left\{ 1 + \frac{0.7}{\mathcal{R} - 1.3} \right\} \frac{\Delta \sigma}{\sigma_t}$$

$$- \frac{1.3}{\mathcal{R} - 1.3} \frac{\Delta t_{R_g}}{t_{R_g}} - \left\{ 1.3 + \frac{1.7}{\mathcal{R} - 1.3} \right\} \frac{\Delta \overline{F}_g}{\overline{F}_g} \qquad \text{(A5-9)}$$

APPENDIX VI

Effect on Air Distance of Curvature of Flight Path

INTRODUCTION:

The air phase begins with the airplane moving parallel to the runway and ends with it climbing at an appreciable angle. A lift in excess of the airplane weight must be produced to bring about this change of the direction of motion, a lift which will result in an increased induced drag. It is shown below that the energy expended in overcoming this induced drag is small enough to make any change in it between test and standard conditions negligible.

We will assume zero wind.

THE EQUATIONS:

Suppose that,

D_h = drag in straight level flight at the appropriate height over the ground

KD_h = induced drag under the above condition

n = normal accelerometer reading

H = height

$\dot{H}$ = dH/dt

V = true speed

H_e = $H + \frac{1}{2}\frac{V^2}{g}$

$\dot{H}_e$ = dH_e/dt

$$\frac{W}{V}\dot{H}_e = \frac{W}{V}\dot{H} + \frac{W}{g}\frac{dV}{dt}$$

$$= F - D_h - (n^2 - 1)KD_h$$

If $\Delta \dot{H}_e$ is the loss in $\dot{H}_e$ when the flight path is curved, i.e. when $n > 1$ if the angle of climb is small

$$\Delta \dot{H}_e = V(n^2 - 1)\frac{KD}{W}$$

For small eventual angles of climb, the total loss ΔH_e in H_e resulting from flying for a while along a curved path is

$$\Delta H_e = \int_0^{\gamma_c} \left(\Delta \dot{H}_e \right) \frac{d\gamma}{\dot{\gamma}}$$

where γ = momentary angle of climb

$\dot{\gamma}$ = $d\gamma/dt$

But $\dot{\gamma} = \frac{n-1}{V} g$

Hence $$\Delta H_e = \int_0^{\gamma_c} \frac{V^2 (n^2-1) K D_h}{(n-1) g W} d\gamma$$

Apart from changes in ground effect $V^2 K D_h$ is constant and equal to $\frac{1}{2} V^2_{md} D_{min}$ where V_{md} is the speed for minimum drag D_{min}. Hence, we have,

$$\Delta H_e = \frac{1}{2} \frac{V_{md}^2}{g} \frac{D_{min}}{W} \int_0^{\gamma_c} (n+1) \, d\gamma$$

$$< \frac{1}{2} \frac{V_{md}^2}{g} \frac{D_{min}}{W} \left(n_{max} + 1 \right) \gamma_c$$

Approximate Amount of Loss: To make a rough assessment of ΔH_e let us make the following assumptions:

a. V_{md} equal to the take-off speed V_T

b. n_{max} less than 1.4

Then $$\Delta H_e < 1.2 \frac{V_T^2}{g} \frac{D_{min}}{W} \gamma_c$$

The excess thrust power available to supply this loss, will not be less than $WV\gamma_c$. The additional air distance will therefore be less than

$$\left\{ W \Delta H_e / W V \gamma_c \right\} \times V$$

$$= \frac{\Delta H_e}{\gamma_c} < 1.2 \frac{V_T^2}{g} \frac{D_{min}}{W}$$

This distance is noticeable but not long. For example, with a V_T of 200 ft./sec and $(D_{min}/W) = 0.1$ it would be about 150 ft. For a light airplane with a V_T of about 60, it would be about 15 ft. It is, therefore, legitimate when standardizing air distances, to omit separate consideration of this distance.

APPENDIX VII

Summary of Correction Formulae

INTRODUCTION:

A summary is presented below, of the correction formulae proposed in the main text. This summary is an outline only, and is not intended as a fully detailed routine.

NOTATION:

Symbol	Definition
$\overline{F}$	mean thrust
$\Delta \overline{F}_b$	correction to mean thrust of basic propulsive systems (used in standardizing JATO take-offs)
$\overline{F}_J$	mean jet thrust of turbo propeller engines
$\overline{F}_p$	mean propeller thrust
F_R	JATO thrust
$\overline{F}_R$	effective mean JATO thrust for phase
N	engine speed
P	brake power to propellers
$\mathcal{R}$	$\overline{F}/\overline{F}_R$
S	length of phase
S_a	length of air phase
S_g	length of ground roll
S_R	distance covered in phase with ATO operating
S_o	length of phase corrected for wind and runway slope
S_W	length of phase with wind and sloping runway

NOTATION (CONTINUED):

Symbol	Definition
t_a	duration of air phase
T_a	ambient absolute air temperature
t_{R_a}	duration of JATO in air phase
t_{R_g}	duration of JATO in ground phase
V_T	true ground speed at take-off
V_{50}	true ground speed at 50 ft.
w	headwind
W	gross weight
σ	relative density = actual density/standard S_I density
$\sin \phi$	slope of runway (positive uphill)

PRELIMINARY CORRECTIONS:

Corrections are first made for wind, and runway slope. Corrections to constant C_L will not be made as a routine, and will therefore, be omitted here. They are detailed in Section 8 of the main text.

Corrections of the ground roll for wind, and runway slope, may be made using the relation

$$S_{g_{t_0}} = S_{g_{t_w}} \left(1 + \frac{w}{V_T}\right)^{1.85} \Bigg/ \left(1 + \frac{2g\, S_{g_T}}{V_{T_T}^2} \sin \phi\right) \tag{A7-1}$$

Correction of the air phase is made by adding the drift

$$S_{a_{t_0}} = S_{a_{t_w}} + w\, t_{a_t} \tag{A7-2}$$

MAIN CORRECTIONS:

Jet Airplanes - Mechanical Computing: In this case, equations (4-1) and

(4-4) will commonly be used. These are

$$\frac{S_{g_s}}{S_{g_t}} = \frac{W_s}{W_t}\,\frac{\sigma_t}{\sigma_s} \Bigg/ \left\{ \frac{2g\,S_{g_t}}{W_t\,V^2_{T_t}}\left(\frac{W_t}{W_s}\,\overline{F}_s - \overline{F}_t\right) + 1 \right\} \qquad \text{(A7-3)}$$

and

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\,\frac{\sigma_t}{\sigma_s}\,h_{v_t} + 50\right) \Bigg/ \left(h_{v_t} + 50 + \frac{S_{a_t}\overline{F}_s}{W_s} - \frac{S_{a_t}\overline{F}_t}{W_t} \right) \qquad \text{(A7-4)}$$

It may usually be assumed that $\overline{F}$ = 0.94 x static thrust. Intakes giving a very bad static performance should, however, be given special consideration.

<u>The General Case</u>: For mixed systems not dominantly propeller driven, and jet airplanes when mechanical computing is not used, the general equations (4-3) or (4-3a) and (4-5) or (4-5a) will be used. With the constants proposed, these become

$$\frac{\Delta S_g}{S_{g_t}} = 2.1\,\frac{\Delta W}{W_t} - \frac{\Delta\sigma}{\sigma_t} - 1.3\,\frac{\Delta\overline{F}}{\overline{F}_t} \qquad \text{(A7-5)}$$

or alternatively

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2.1}\left(\frac{\sigma_s}{\sigma_t}\right)\left(\frac{\overline{F}_s}{\overline{F}_t}\right)^{-1.3} \qquad \text{(A7-5a)}$$

and

$$\frac{\Delta S_a}{S_{a_t}} = 2.3\,\frac{\Delta W}{W_t} - .07\,\frac{\Delta\sigma}{\sigma_t} - 1.6\,\frac{\Delta\overline{F}}{\overline{F}_t} \qquad \text{(A7-6)}$$

or alternatively

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{2.3}\left(\frac{\sigma_s}{\sigma_t}\right)^{-0.7}\left(\frac{\overline{F}_s}{\overline{F}_t}\right)^{-1.6} \qquad \text{(A7-6a)}$$

For moderate corrections either type of equation is satisfactory, but if the corrections are large (for example, $|\Delta S/S| < 0.2$) the exponential forms will be appreciably more accurate.

Fixed Pitch Propellers: Corrections may be at constant engine speed or at full throttle here. At constant engine speed we use equations (10-7) and (10-8), that is

$$\frac{\Delta S_g}{S_{g_t}} = 2.4\,\frac{\Delta W}{W_t} - 2.4\,\frac{\Delta\sigma}{\sigma_t} \tag{A7-7}$$

and

$$\frac{\Delta S_a}{S_{a_t}} = 2.2\,\frac{\Delta W}{W_t} - 2.2\,\frac{\Delta\sigma}{\sigma_t} \tag{A7-8}$$

If $\Delta S/S$ is numerically large, it is again preferable to use the exponential forms

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2.4}\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.4} \tag{A7-7a}$$

and

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{2.2}\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.2} \tag{A7-8a}$$

At full throttle we will have a correction to engine speed (equation 7-4).

$$\frac{\Delta N}{N_t} = \frac{1}{4}\,\frac{\Delta T_a}{T_{a_t}} \tag{A7-9}$$

and also equations (10-9) and (10-10).

$$\frac{\Delta S_g}{S_{g_t}} = 2.4\,\frac{\Delta W}{W_t} - 2.4\,\frac{\Delta\sigma}{\sigma_t} + 0.5\,\frac{\Delta T_a}{T_{a_t}} \tag{A7-10}$$

and

$$\frac{\Delta S_a}{S_{a_t}} = 2.2\,\frac{\Delta W}{W_t} - 2.2\,\frac{\Delta\sigma}{\sigma_t} + 0.6\,\frac{\Delta T_a}{T_{a_t}} \tag{A7-11}$$

with the corresponding exponential forms

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2.4}\left(\frac{\sigma_s}{\sigma_t}\right)^{-2.4}\left(\frac{T_{a_s}}{T_{a_t}}\right)^{0.5} \tag{A7-10a}$$

$$\frac{S_{as}}{S_{at}} = \left(\frac{W_s}{W_t}\right)^{2.3} \left(\frac{\sigma_s}{\sigma_t}\right)^{-2.2} \left(\frac{T_{as}}{T_{at}}\right)^{0.6} \tag{A7-11a}$$

<u>Constant Speed Propellers</u>: This section applies to airplanes which are entirely, or almost entirely, propeller driven at take-off. For the ground roll, we have (equation (10-11) or (10-13).

$$\frac{\Delta S_g}{S_{gt}} = 2.6\,\frac{\Delta W}{W_t} - 1.7\,\frac{\Delta\sigma}{\sigma_t} - 0.7\,\frac{\Delta N}{N_t} - 0.9\,\frac{\Delta P}{P_t} \tag{A7-12}$$

and the alternative form

$$\frac{S_{gs}}{S_{gt}} = \left(\frac{W_s}{W_t}\right)^{2.6} \left(\frac{\sigma_s}{\sigma_t}\right)^{-1.7} \left(\frac{N_s}{N_t}\right)^{-0.7} \left(\frac{P_s}{P_t}\right)^{-0.9} \tag{A7-12a}$$

For the air phase, we distinguish between light and heavy airplanes. <u>For light airplanes</u>, we have (equation (10-12)

$$\frac{\Delta S_a}{S_{at}} = 2.3\,\frac{\Delta W}{W_t} - 1.2\,\frac{\Delta\sigma}{\sigma_t} - 0.5\,\frac{\Delta N}{N_t} - 1.1\,\frac{\Delta P}{P_t} \tag{A7-13}$$

or alternatively

$$\frac{S_{as}}{S_{at}} = \left(\frac{W_s}{W_t}\right)^{2.3} \left(\frac{\sigma_s}{\sigma_t}\right)^{-1.2} \left(\frac{N_s}{N_t}\right)^{-0.5} \left(\frac{P_s}{P_t}\right)^{-1.1} \tag{A7-13a}$$

<u>For heavy airplanes</u> (equation (10-14)

$$\frac{\Delta S_a}{S_{at}} = 2.6\,\frac{\Delta W}{W_t} - 1.5\,\frac{\Delta\sigma}{\sigma_t} - 0.5\,\frac{\Delta\sigma}{\sigma_t} - 1.1\,\frac{\Delta P}{P_t} \tag{A7-14}$$

or alternatively

$$\frac{S_{as}}{S_{at}} = \left(\frac{W_s}{W_t}\right)^{2.6} \left(\frac{\sigma_s}{\sigma_t}\right)^{-1.5} \left(\frac{N_s}{N_t}\right)^{-0.5} \left(\frac{P_s}{P_t}\right)^{-1.1} \tag{A7-14a}$$

Turbo Propellers: It will probably be found that, equations (A7-10) and (A7-12), or their alternates, may be used for turbo propeller airplanes. However, if this approximation is to be avoided, the data must be standardized using equations (A7-5) and (A7-6). To estimate $\frac{\Delta \overline{F}}{\overline{F}_t}$ use equation (7-6)

$$\frac{\Delta \overline{F}}{\overline{F}_t} = \frac{\Delta \overline{F}_j}{\overline{F}_t} + \frac{\overline{F}_p}{\overline{F}} \frac{\Delta \overline{F}_p}{\overline{F}_p} \qquad (A7\text{-}15)$$

and estimate $\Delta \overline{F}_p/\overline{F}_p$ by equation (7-5), i.e.

$$\frac{\Delta \overline{F}_p}{\overline{F}_{p_t}} = 0.7 \frac{\Delta T}{T_t} + 0.5 \frac{\Delta \sigma}{\sigma_t} + 0.5 \frac{\Delta N}{N_t} - 0.2 \frac{\Delta W}{W_t} \qquad (A7\text{-}16)$$

Part-Time Assistance: Again, equations (A7-5) and (A7-6) are used basically, but with an effective mean thrust. We are considering primarily JATO, but the method can be applied to other forms of thrust boost operated over a limited period.

The test effective, mean thrust boost $\overline{F}_R$ in either the ground roll or air phase given by equation (9-1)

$$\overline{F}_R = \frac{S_R}{S} F_R \qquad (A7\text{-}17)$$

where F_R = JATO thrust

S_R = distance covered in phase with JATO operating

S = total length of phase

The standard effective mean thrust in the air phase is either zero (ATO to cease at take-off) or equal to the actual ATO thrust (ATO to last to 50 ft.). The standard effective mean thrust in the ground roll, however, depends on the time t_{a_s} during which the ATO is to operate in the air phase under standard conditions.

We have (equation (9-2)

$$t_{a_s} = 0 \quad \text{ATO ceasing at take-off}$$

$$= 2S_{a_s}/(V_{s_T} + V_{s_{50}}) \quad \text{for ATO ceasing at 50 ft.} \qquad (A7\text{-}18)$$

Hence if t_{a_t} was the test duration of the ATO in the air phase, we must correct the ATO duration in the ground roll by (equation (9-3).

$$\Delta t_{R_g} = t_{R_{a_t}} - t_{R_{a_s}} \qquad \text{(A7-19)}$$

The correction to the air phase is then given by equation (A7-6) using the total mean effective thrust. For the ground roll, however, we use equation (9-4)

$$\begin{aligned} \frac{\Delta S_g}{S_{g_t}} = & \left\{2.3 - \frac{2.3}{R - 1.3}\right\} \frac{\Delta W}{W_t} \\ & - \left\{1 + \frac{0.7}{R - 1.3}\right\} \frac{\Delta \sigma}{\sigma_t} \\ & - \frac{1.3}{R - 1.3} \frac{\Delta t_{R_g}}{t_{R_{g_t}}} \\ & - \left\{1.3 + \frac{1.7}{R - 1.3}\right\} \frac{\Delta \overline{F}_r}{\overline{F}_{r_t}} \end{aligned} \qquad \text{(A7-20)}$$

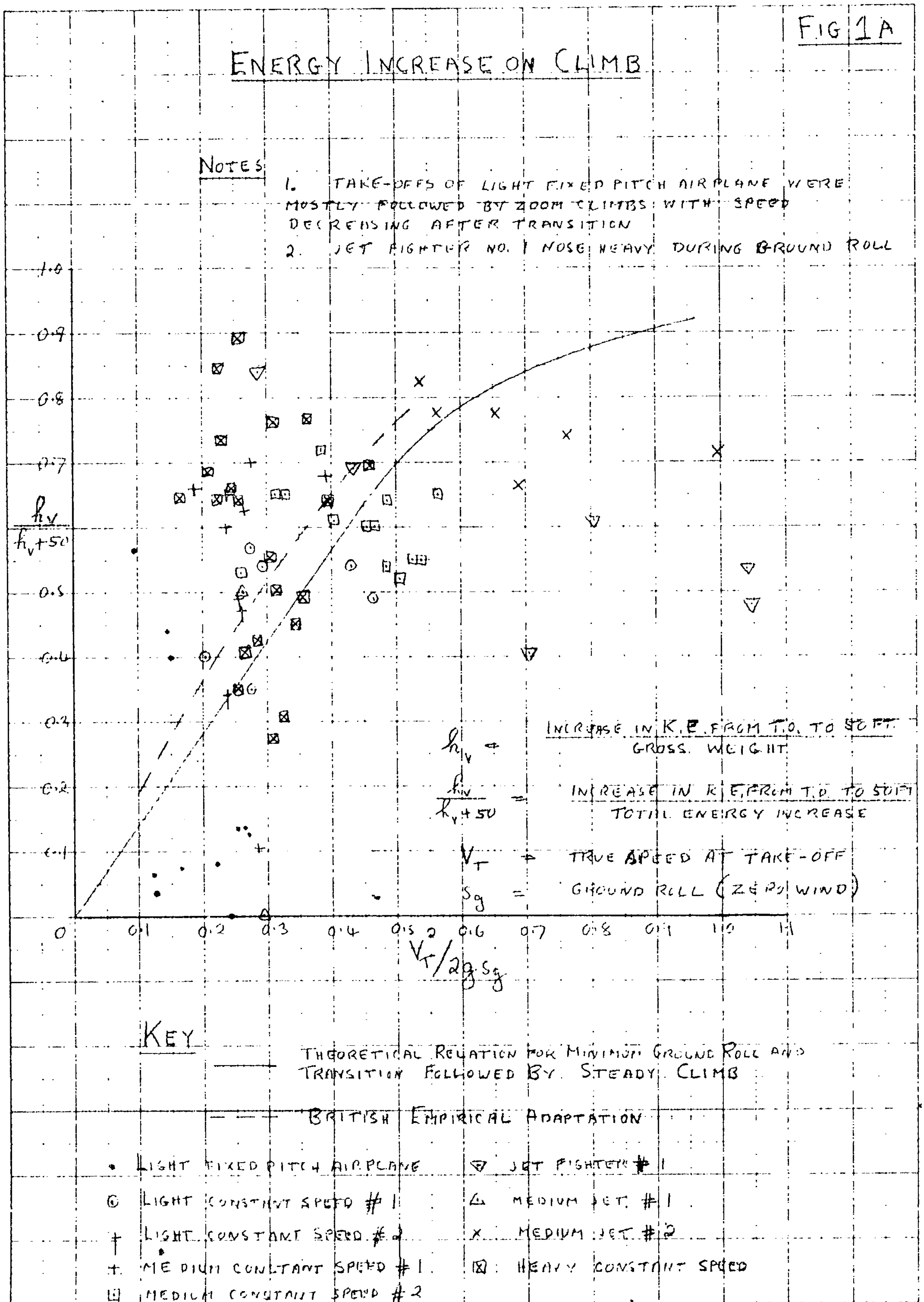


FIG 1A
ENERGY INCREASE ON CLIMB
NOTES
1. TAKE-OFFS OF LIGHT FIXED PITCH AIRPLANE WERE MOSTLY FOLLOWED BY ZOOM CLIMBS WITH SPEED DECREASING AFTER TRANSITION
2. JET FIGHTER NO. 1 NOSE HEAVY DURING GROUND ROLL
1.0
0.9
0.8
0.7
$\frac{h_v}{h_v+50}$
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
$V_T^2/2g.S_g$
h_v = INCREASE IN K.E. FROM T.O. TO 50 FT. / GROSS WEIGHT
$\frac{h_v}{h_v+50}$ = INCREASE IN K.E. FROM T.O. TO 50 FT / TOTAL ENERGY INCREASE
V_T = TRUE SPEED AT TAKE-OFF
S_g = GROUND ROLL (ZERO WIND)
KEY
THEORETICAL RELATION FOR MINIMUM GROUND ROLL AND TRANSITION FOLLOWED BY STEADY CLIMB
BRITISH EMPIRICAL ADAPTATION
LIGHT FIXED PITCH AIRPLANE
LIGHT CONSTANT SPEED #1
LIGHT CONSTANT SPEED #2
MEDIUM CONSTANT SPEED #1
MEDIUM CONSTANT SPEED #2
JET FIGHTER #1
MEDIUM JET #1
MEDIUM JET #2
HEAVY CONSTANT SPEED

FIG. 1B

TECHNIQUES OF CLIMB TO 50 FT

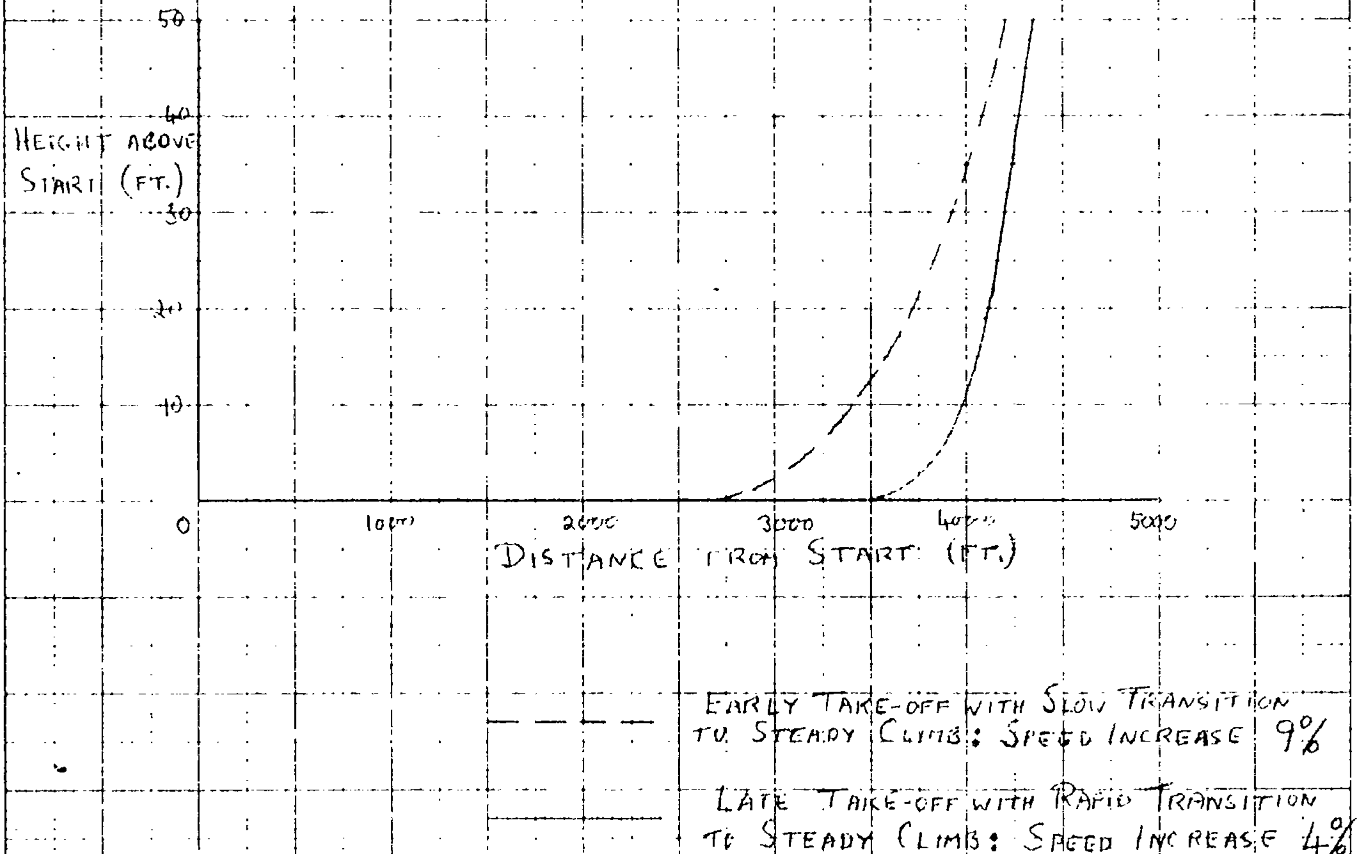
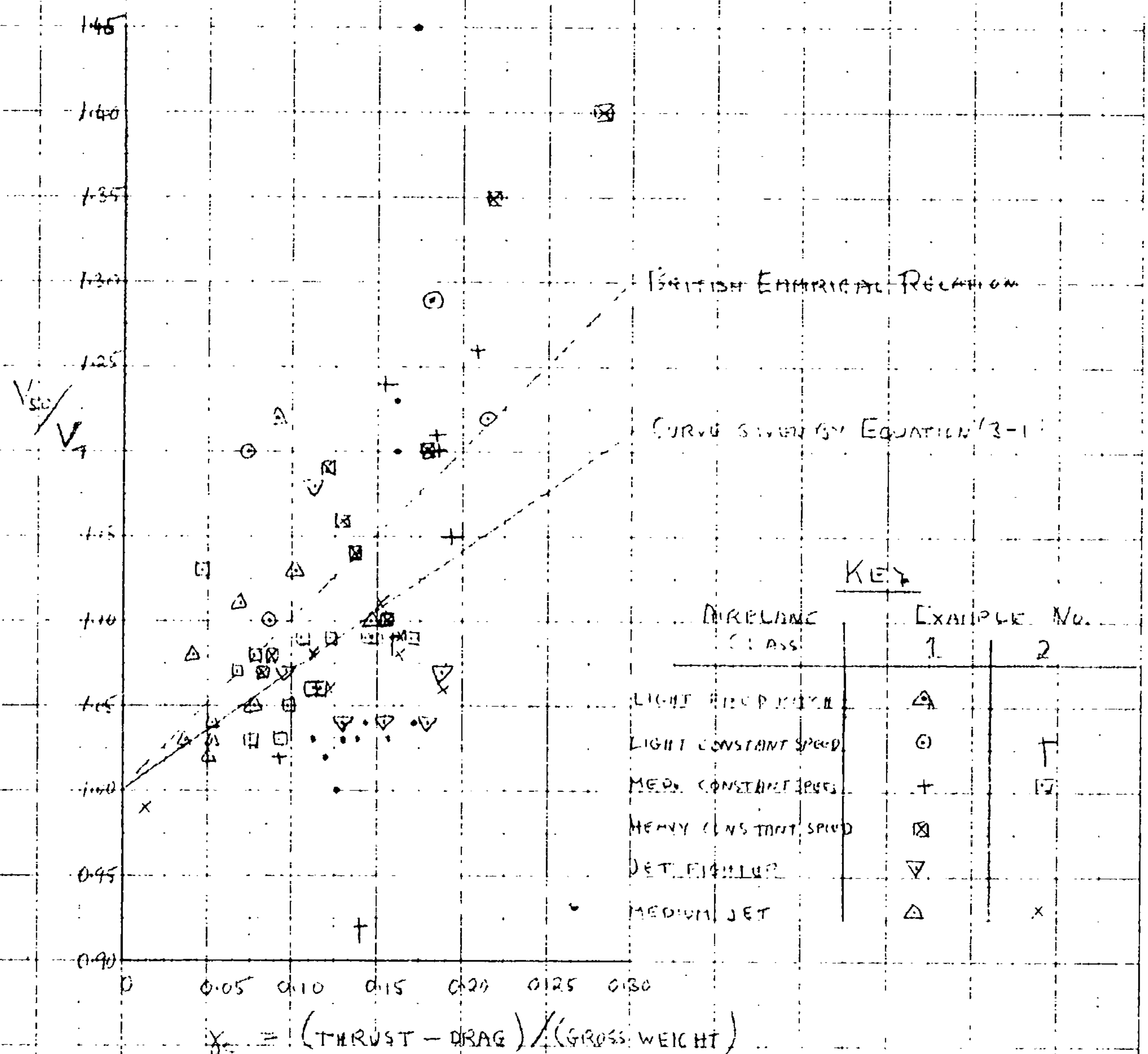


FIG 2

SPEED CHANGE DURING AIR PHASE



NOTES

1. MOST TAKE-OFFS OF LIGHT FIXED PITCH AIRPLANE ENDED WITH ZOOM CLIMB WITH SPEED DECREASING.

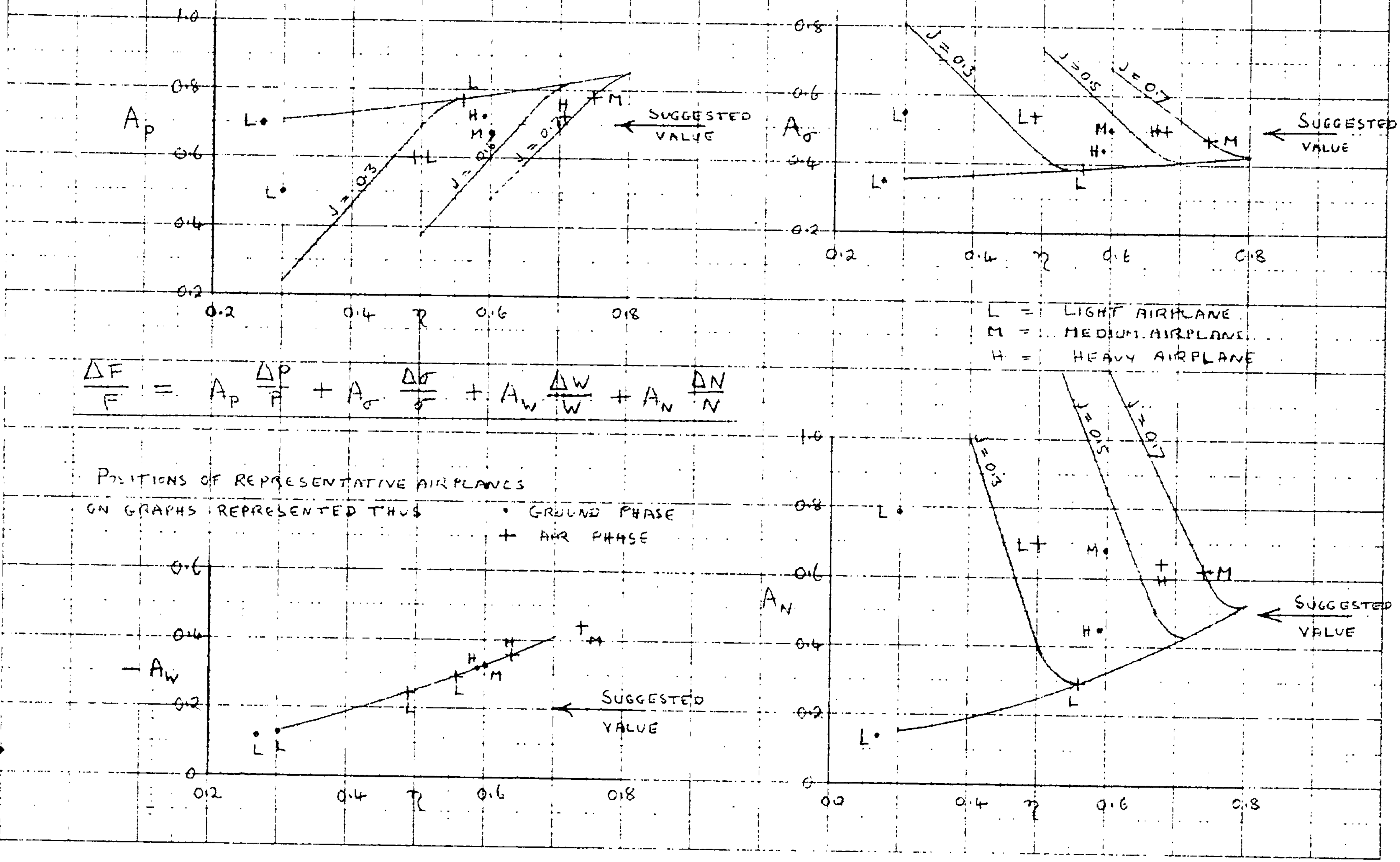
2. JET FIGHTER NOSE HEAVY DURING GROUND ROLL

3. V_{50}/V_T = (SPEED AT 50 FT)/(SPEED AT TAKE-OFF)

STANDARDIZATION OF PROPELLER THRUST

CHANGE OF THRUST WITH POWER, DENSITY, WEIGHT AND RPM

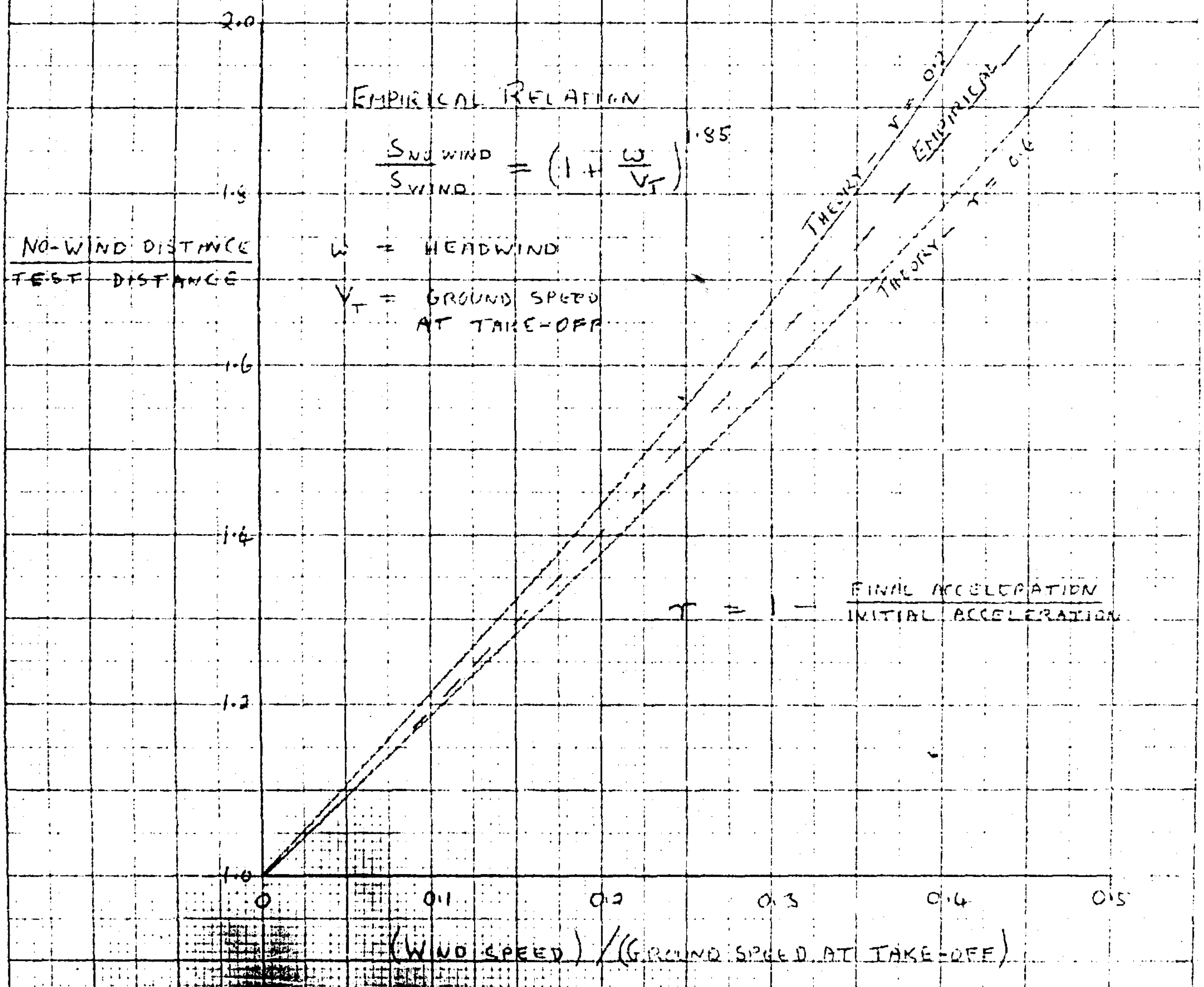
FIG. 3



$$\frac{\Delta F}{F} = A_p \frac{\Delta P}{P} + A_\sigma \frac{\Delta \sigma}{\sigma} + A_W \frac{\Delta W}{W} + A_N \frac{\Delta N}{N}$$

FIG. 4

WIND CORRECTION TO GROUND ROLL



CHECK OF WEIGHT AND PRESSURE CORRECTIONS

LIGHT AIRPLANE WITH FIXED PITCH PROPELLER

FIG 5

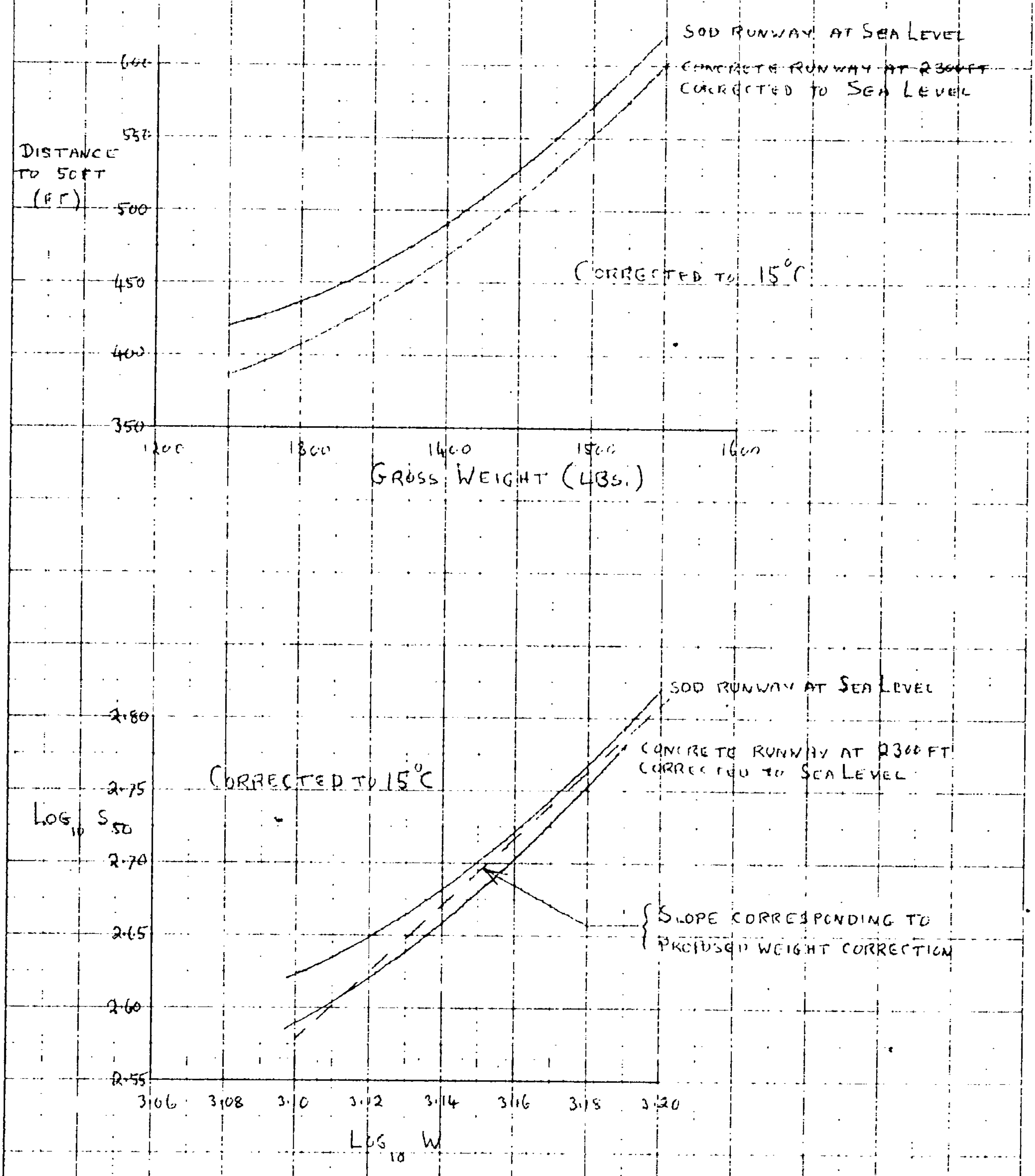
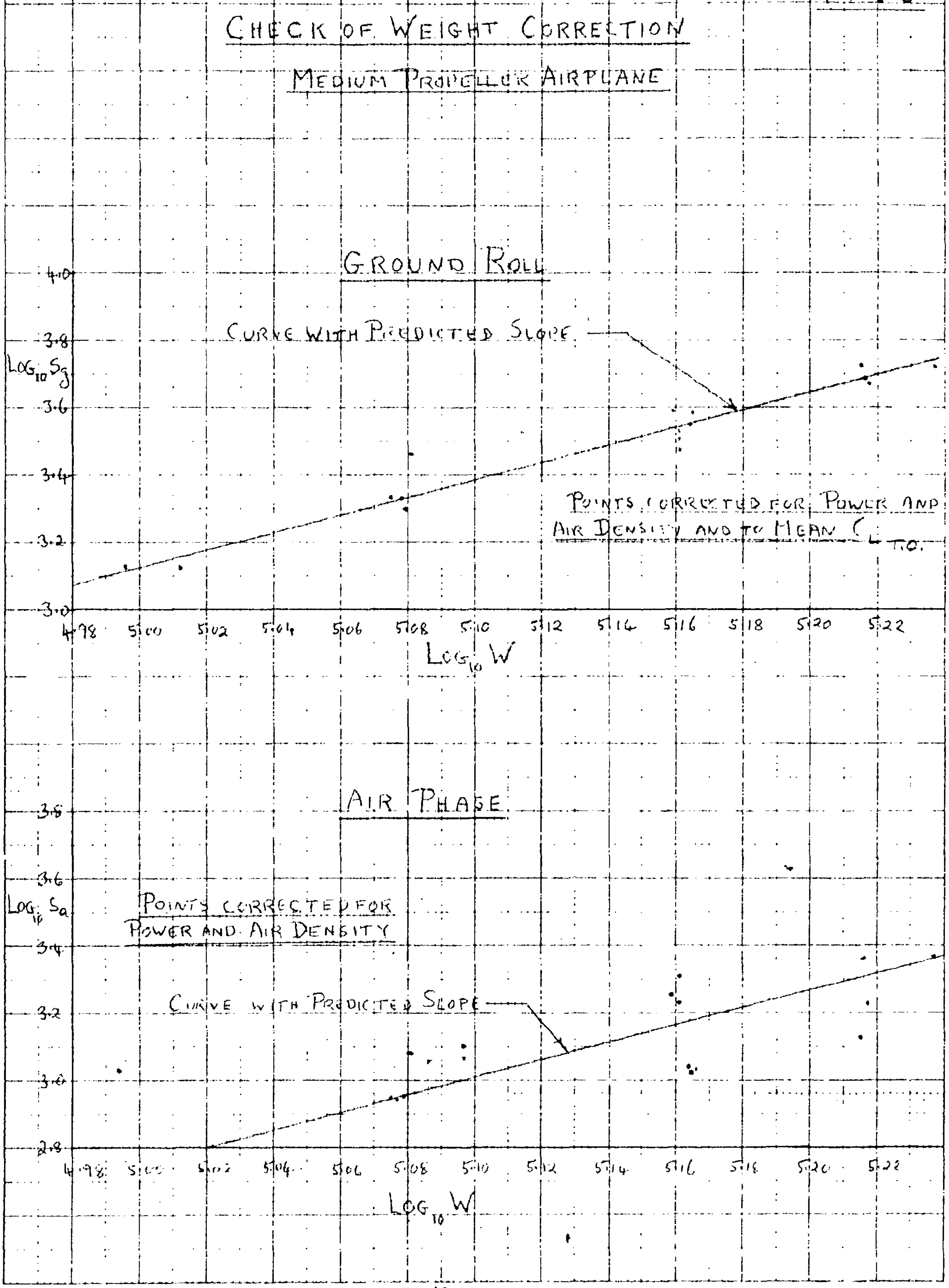


FIG. 6



CHECK OF WEIGHT CORRECTION
MEDIUM PROPELLER AIRPLANE
GROUND ROLL
CURVE WITH PREDICTED SLOPE
POINTS CORRECTED FOR POWER AND AIR DENSITY AND TO MEAN $C_{L\,T.O.}$
$\mathrm{LOG}_{10} S_g$
4.0
3.8
3.6
3.4
3.2
3.0
4.98
5.00
5.02
5.04
5.06
5.08
5.10
5.12
5.14
5.16
5.18
5.20
5.22
$\mathrm{LOG}_{10} W$
AIR PHASE
POINTS CORRECTED FOR POWER AND AIR DENSITY
CURVE WITH PREDICTED SLOPE
$\mathrm{LOG}_{10} S_a$
3.8
3.6
3.4
3.2
3.0
2.8
$\mathrm{LOG}_{10} W$

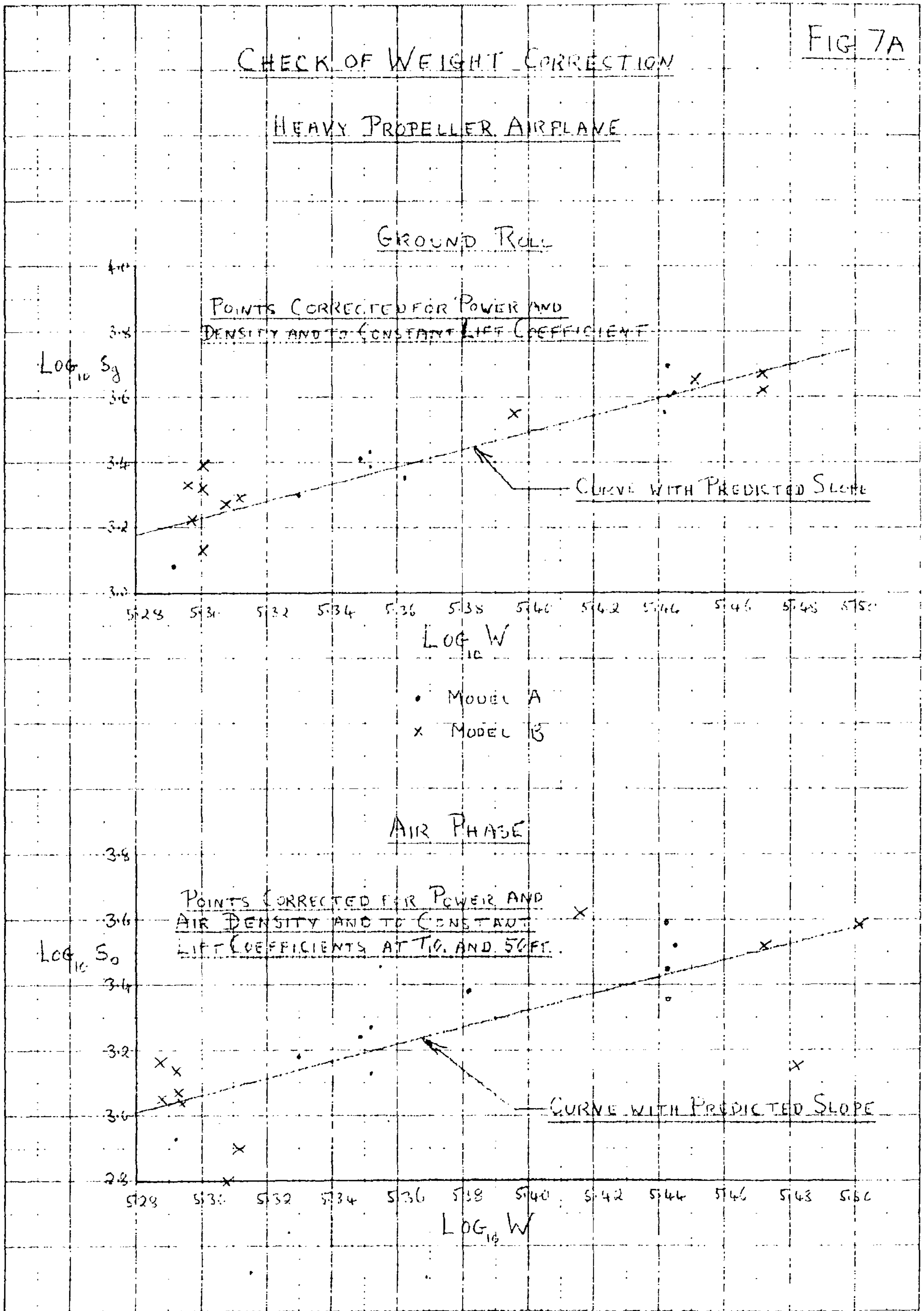
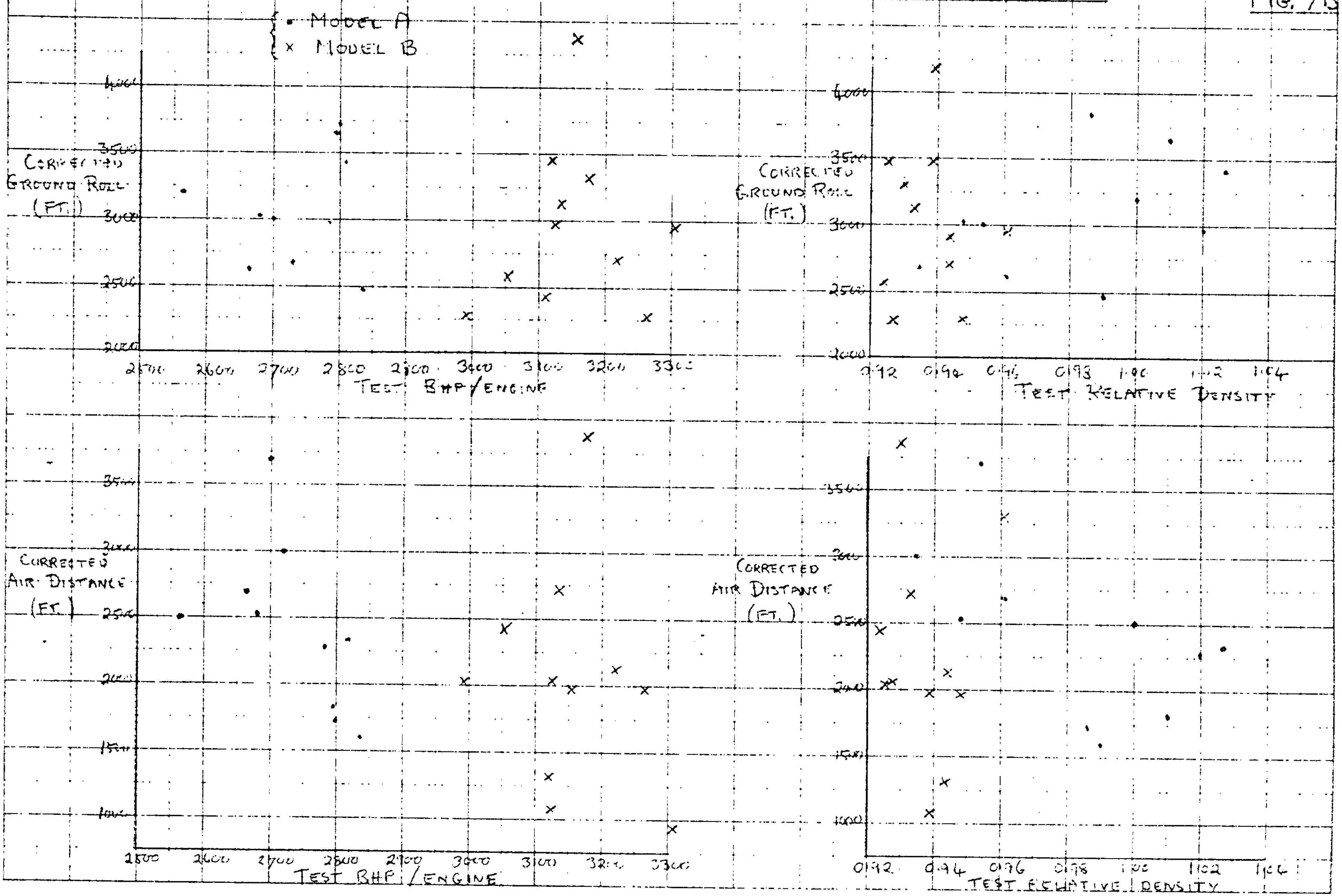


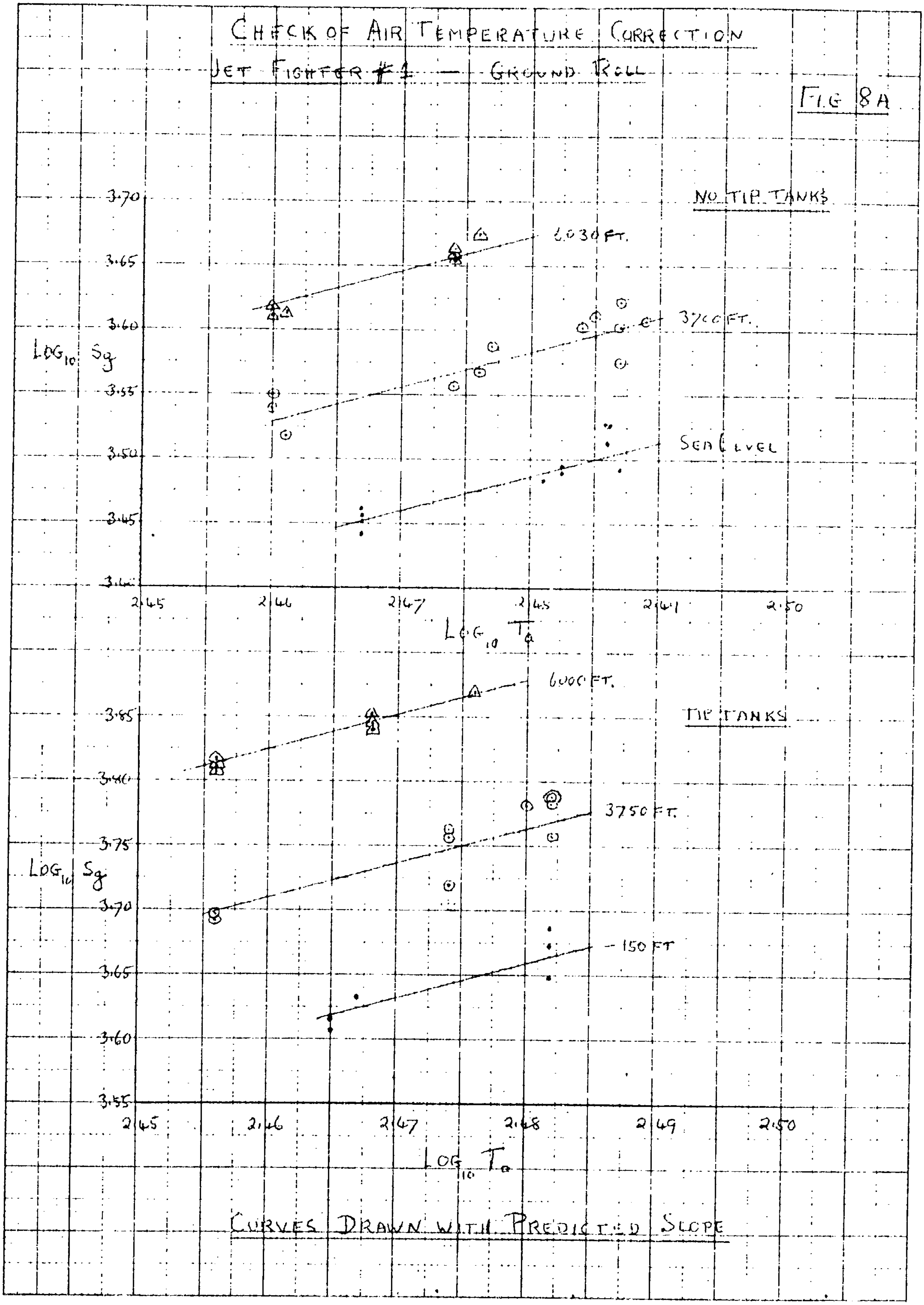
FIG 7A
CHECK OF WEIGHT CORRECTION
HEAVY PROPELLER AIRPLANE
GROUND ROLL
POINTS CORRECTED FOR POWER AND DENSITY AND TO CONSTANT LIFT COEFFICIENT
LOG10 Sg
CURVE WITH PREDICTED SLOPE
LOG10 W
• MODEL A
× MODEL B
AIR PHASE
POINTS CORRECTED FOR POWER AND AIR DENSITY AND TO CONSTANT LIFT COEFFICIENTS AT T.O. AND 50 FT.
LOG10 Sa
CURVE WITH PREDICTED SLOPE
LOG10 W

CHECK OF POWER AND DENSITY CORRECTIONS – HEAVY PROPELLER AIRPLANE

FIG. 7B



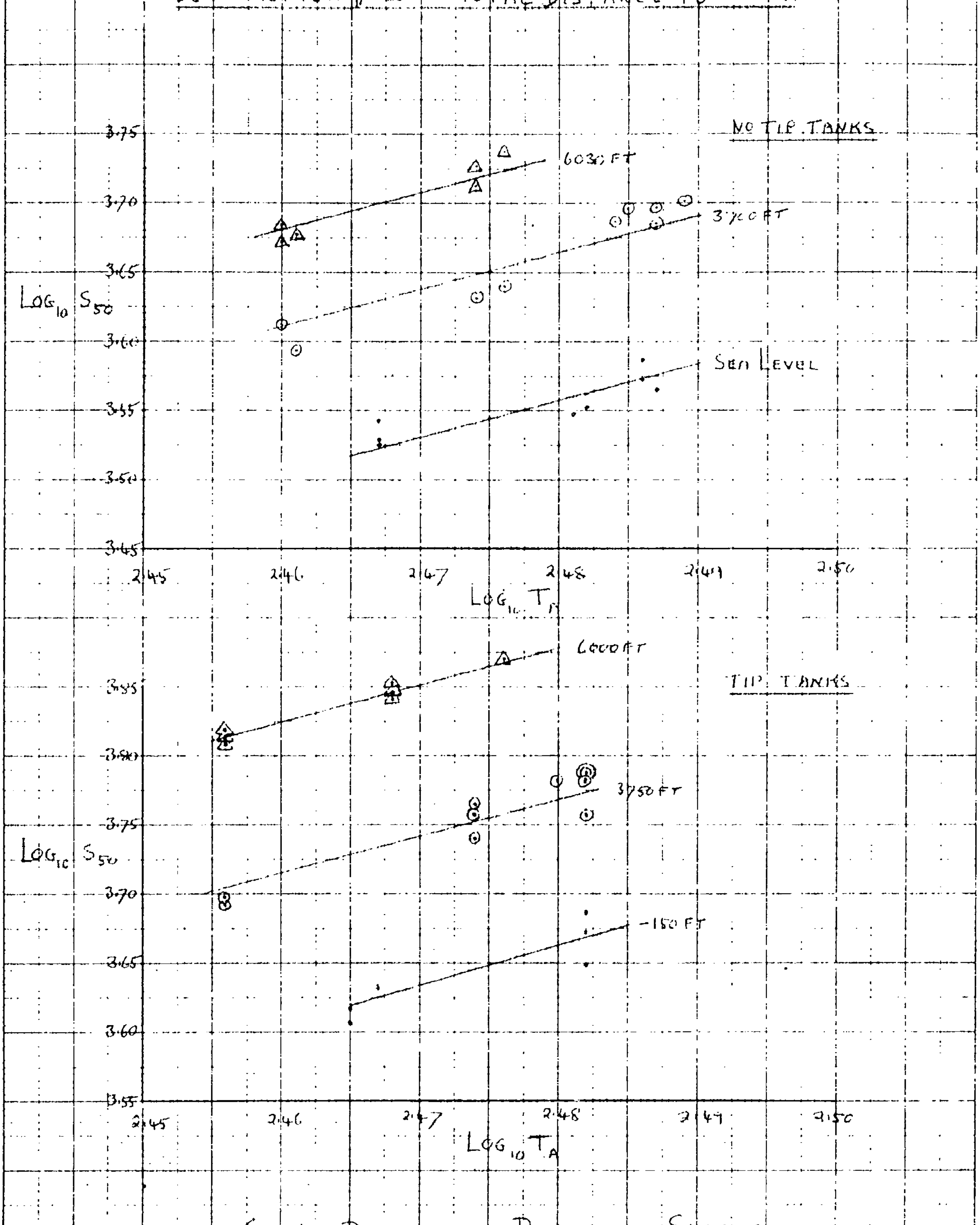
ALL POINTS CORRECTED FOR POWER, DENSITY, WEIGHT, AND TO CONSTANT C, AT T.O. AND 50 FT


CHECK OF AIR TEMPERATURE CORRECTION
JET FIGHTER #1 — GROUND ROLL
FIG 8A
NO TIP TANKS
6030 FT.
3700 FT.
SEA LEVEL
LOG10 Sg
3.70
3.65
3.60
3.55
3.50
3.45
3.40
2.45
2.46
2.47
2.48
2.49
2.50
LOG10 Ta
6000 FT.
TIP TANKS
3750 FT.
-150 FT
3.85
3.80
3.75
3.70
3.65
3.60
3.55
CURVES DRAWN WITH PREDICTED SLOPE

CHECK OF AIR TEMPERATURE CORRECTION

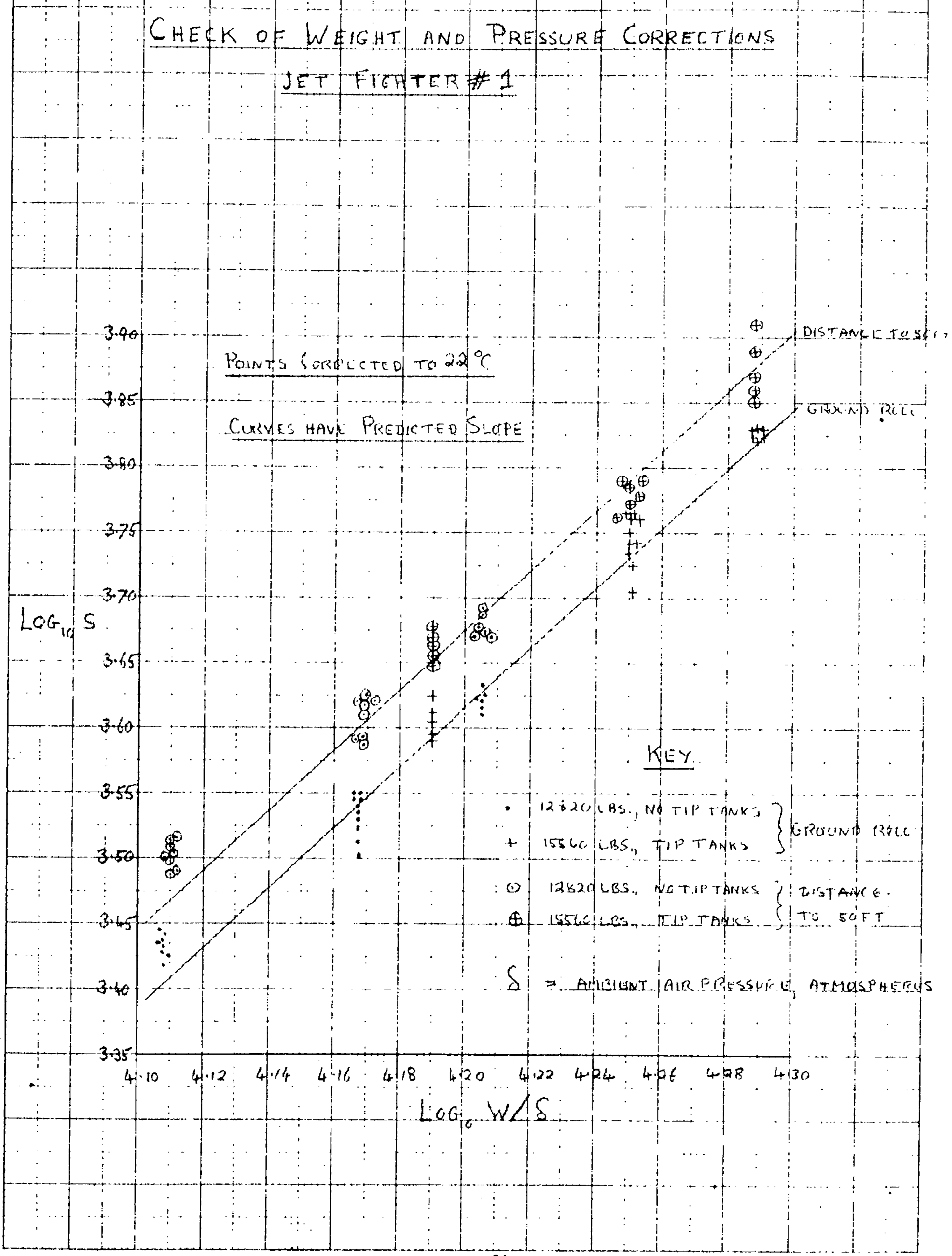
JET FIGHTER #1 — TOTAL DISTANCE TO 50 FT

FIG. 8B



CURVES DRAWN WITH PREDICTED SLOPES

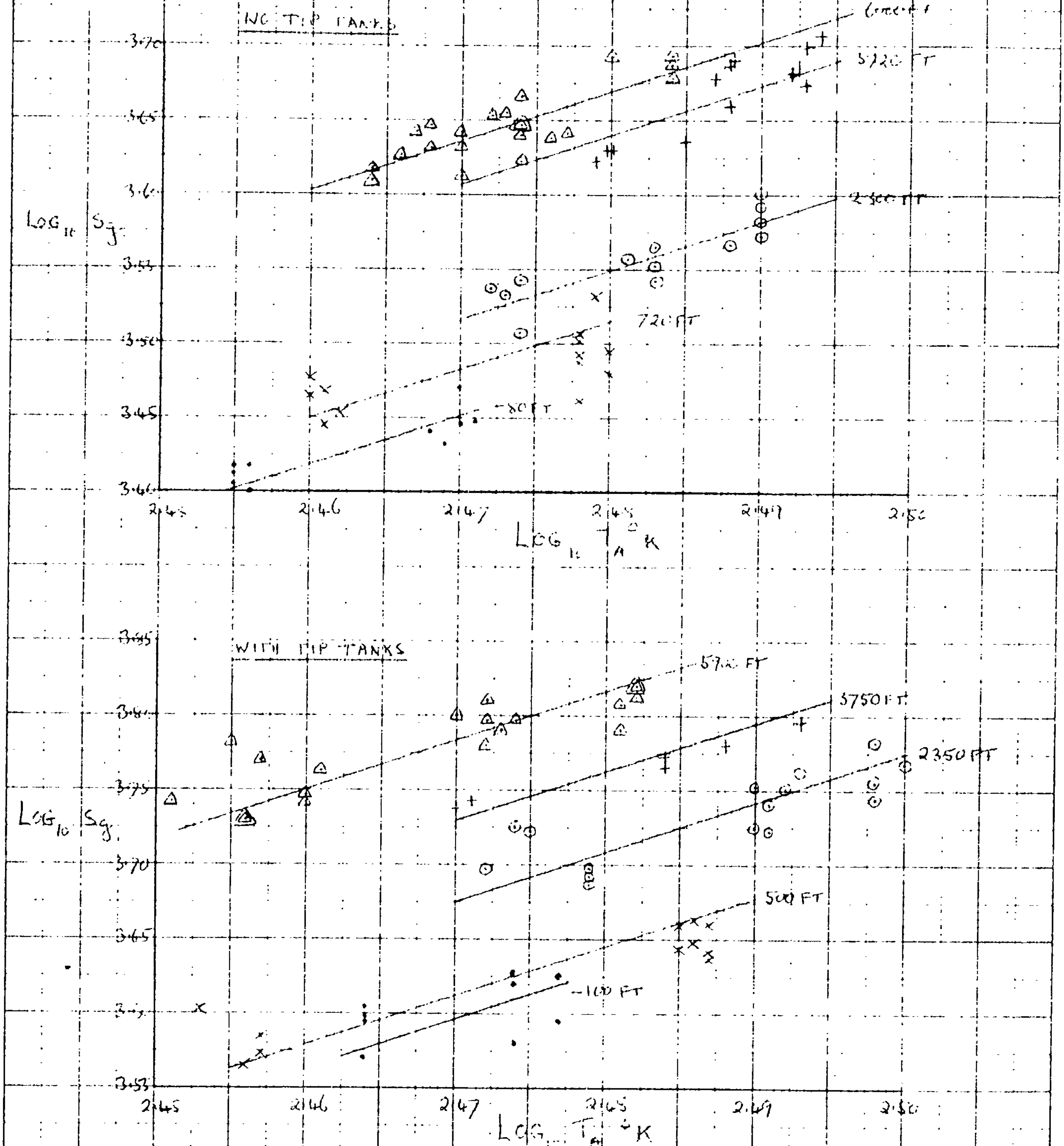
FIG. 8C


CHECK OF WEIGHT AND PRESSURE CORRECTIONS
JET FIGHTER #1
POINTS CORRECTED TO 22°C
CURVES HAVE PREDICTED SLOPE
DISTANCE TO 50FT
GROUND ROLL
3.90
3.85
3.80
3.75
3.70
3.65
3.60
3.55
3.50
3.45
3.40
3.35
LOG10 S
4.10
4.12
4.14
4.16
4.18
4.20
4.22
4.24
4.26
4.28
4.30
LOG10 W/δ
KEY
12820 LBS., NO TIP TANKS
15560 LBS., TIP TANKS
GROUND ROLL
12820 LBS., NO TIP TANKS
15560 LBS., TIP TANKS
DISTANCE TO 50 FT
δ = AMBIENT AIR PRESSURE, ATMOSPHERES

CHECK OF AIR TEMPERATURE CORRECTION

JET FIGHTER # 2 — GROUND ROLL

FIG 9A

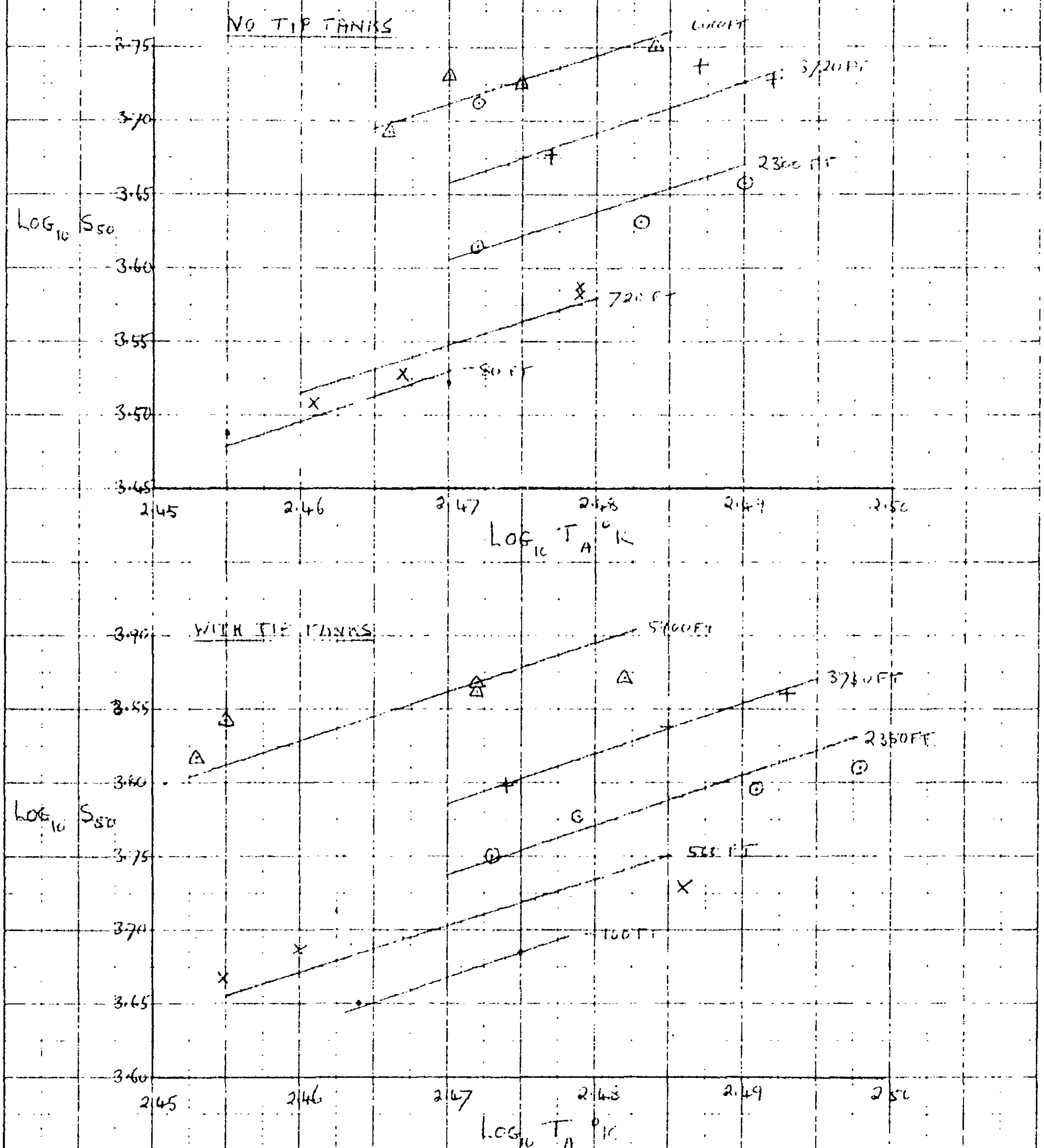


CURVES DRAWN WITH PREDICTED SLOPE

CHECK ON TEMPERATURE CORRECTION

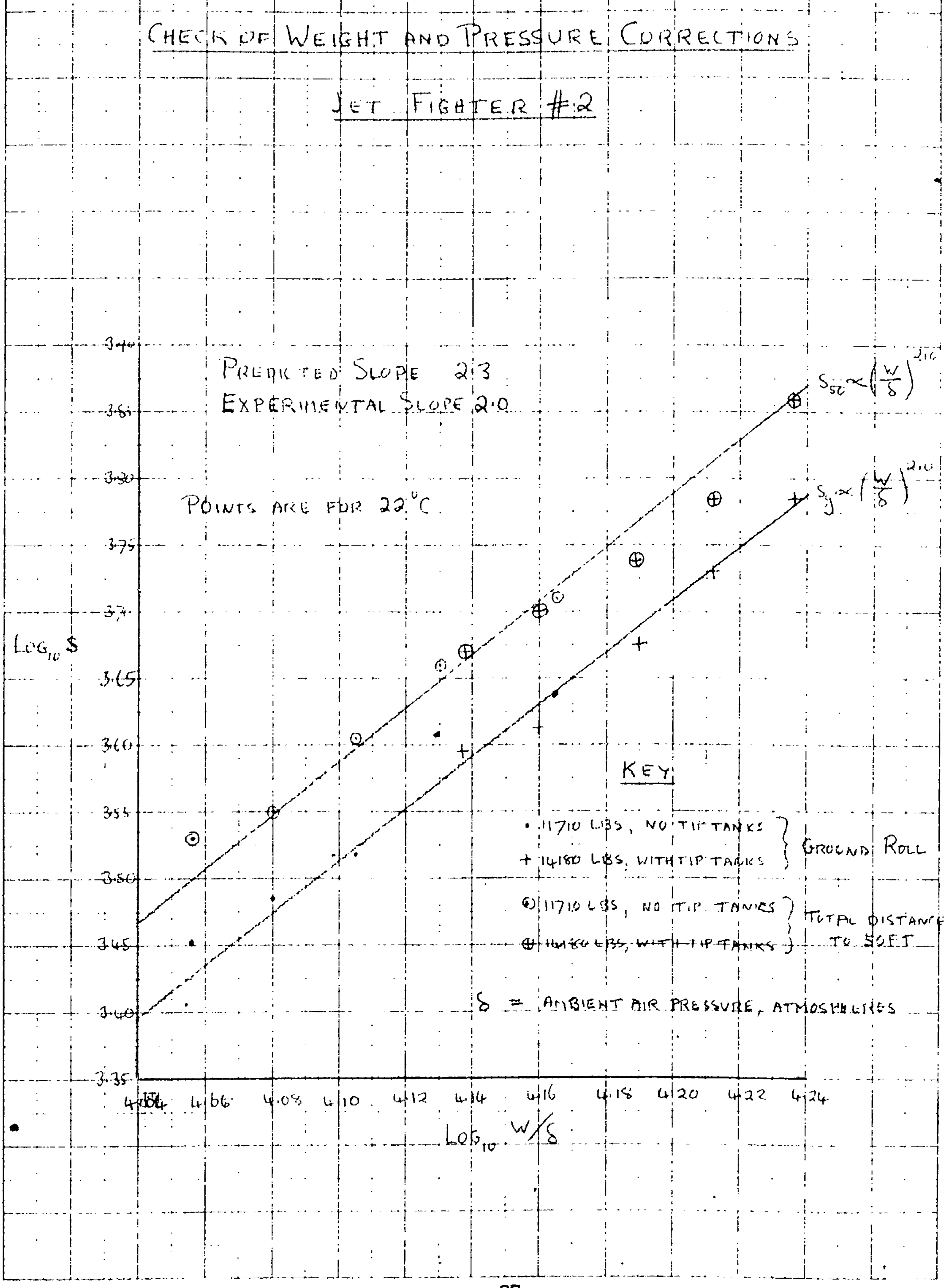
JET FIGHTER #2 – TOTAL DISTANCE TO 50FT.

FIG 9B



CURVES DRAWN WITH PREDICTED SLOPE

Fig 9 C



CHECK OF WEIGHT AND PRESSURE CORRECTIONS
JET FIGHTER #2
PREDICTED SLOPE 2.3
EXPERIMENTAL SLOPE 2.0
POINTS ARE FOR 22°C
KEY
• 11710 LBS, NO TIP TANKS
+ 14180 LBS, WITH TIP TANKS
GROUND ROLL
⊙ 11710 LBS, NO TIP TANKS
⊕ 14180 LBS, WITH TIP TANKS
TOTAL DISTANCE TO 50 FT
δ = AMBIENT AIR PRESSURE, ATMOSPHERES
LOG₁₀ S
LOG₁₀ W/δ

FIG. 10

VARIATION OF ACCELERATION WITH SPEED

REPRESENTATIVE EXAMPLES